

Name \_\_\_\_\_

1. Consider the three vectors  $\mathbf{u} = (1,3,2)$ ,  $\mathbf{v} = (3,1,-8)$  and  $\mathbf{w} = (2,2,1)$  in  $\mathbb{R}^3$ .
  - a. Find the cosine of the angle between vectors  $\mathbf{u}$  and  $\mathbf{v}$ .
  
  
  
  
  
  
  
  
  
  
  - b. Find the distance between  $\mathbf{u}$  and  $\mathbf{v}$ .
  
  
  
  
  
  
  
  
  
  
  - c. Find a vector of magnitude 24 in the direction of  $-\mathbf{w}$ . (Hint: Find a unit vector first.)
  
  
  
  
  
  
  
  
  
  
  - d. Demonstrate the Cauchy Schwartz inequality for  $\mathbf{u}$  and  $\mathbf{v}$ .
  
  
  
  
  
  
  
  
  
  
  - e. Demonstrate the triangular inequality for  $\mathbf{u}$  and  $\mathbf{v}$ .
  
  
  
  
  
  
  
  
  
  
  - f. Demonstrate the Pythagorean Theorem using  $\mathbf{v}$  and  $\mathbf{w}$ .

2. Consider the orthogonal basis  $B = \{ (5,0,12,0), (0,3,0,-4), (-12,0,5,0), (0,4,0,3) \}$  of  $\mathbb{R}^4$ .
- a. Find an orthonormal basis  $D$  for  $\mathbb{R}^4$ .
- b. Find the coordinates  $[\mathbf{x}]_D$  relative to the orthonormal basis  $D$  for  $\mathbf{x} = (26,20,-13,10)$  given relative to the standard basis  $S$  using the methods from chapter 4 involving the augmented matrix,  $[D \text{ col} \mid [\mathbf{x}]_S]$ .
- c. Find the coordinates  $[\mathbf{x}]_D$  relative to the orthonormal basis  $D$  for  $\mathbf{x} = (26,20,-13,10)$  given relative to the standard basis  $S$  using the methods from chapter 5 involving the Fourier coefficients of  $\mathbf{x}$  relative to  $D$ .

5. a. Give an example of an inner product defined on the vector space of trig functions on the interval  $[0, \pi/4]$ .
- b. Evaluate the inner product  $\langle \sin x, \cos x \rangle$  in the space you defined above. (In case your calculus is a little rusty, letting  $u = \sin x$  might be a helpful substitution somewhere in your evaluation of the inner product.
6. Find the angle between the diagonal of a cube and one of its edges.

7. a. Use the Gram-Schmidt process to find an orthogonal basis for the vector space whose basis is  $\{(0, 1, 2), (1, 3, 1), (1, 1, 1)\}$ .
- b. Convert the basis  $W$  into an orthonormal basis and name it  $U = \{u_1, u_2, u_3\}$ .
- c. Write  $U$  in columns and call the matrix  $U_{\text{col}}$ . Evaluate the matrix product  $U_{\text{col}} \text{ times } U_{\text{col}}^T$  or  $U_{\text{col}}^T \text{ times } U_{\text{col}}$  and explain why one of the results shows whether  $U$  is an orthonormal basis for the vector space.

8. State and prove the triangular inequality.

9. Use the inner product as an example to discuss the concept of generalization in mathematics. Add some items not in the class notes to compliment your discussion.