Name ___________________________________

1. Tell whether this function is a linear transformation. (yes or no)

   T(x,y) = (2x + 5, 4y - 3y)

2. Give the following standard matrices for linear transformations from R^2 ----> R^2 where

   a. A_1 produces an expansion (stretching) in the x direction by a factor of 2 and in the y direction by 5.

   b. A_2 produces a reflection through the y axis (in the x direction).

   c. A_3 produces a rotation of 30 degrees about the origin.

   d. A_4 produces a horizontal shear by a factor of 3 times the y coordinate.

   f. Give the matrix A that will produce the reflection in part b, followed by the rotation in part c, followed by the shear in part d. Just use the names in the answer.

3. Consider the linear transformation given by the "rule", $T(x,y,z) = (x+3y+2z, 4x-y+5z)$,
a. Give bases for the following.

domain of T

codomain of T

range of T

kernel of T

rank of T

nullity of T

b. Find the image of ((3,2,1).

c. Find the pre-image of (5,27).

d. Write T in terms of its effect on the standard basis.

e. Write T in terms of the standard transformation matrix.

f. Give reasons for your determination as to whether T is

one-to-one

onto

an isomorphism

g. If T is an isomorphism, write the inverse of T in the form $T^{-1}(x,y,z) = (..,..,..)$

4. Consider the linear transformation given by its effect on the standard basis:
T(1,0,0) = (1,1,3),   T(0,1,0) = (0,1,0),   T(0,0,1) = (2,1,7).
.
a. Give bases for the following.

domain of T

codomain of T

range of T

kernel of T

rank of T

nullity of T

b. Find the image of ((3,2,1).

c. Find the pre-image of (2,8,5).

d. Write T as a rule.

e. Write T in terms of the standard
transformation matrix A.

f. Give reasons for your determination as to whether T is

one-to-one

onto

an isomorphism

g. If T is an isomorphism, write the inverse of
T in the form $T^{-1}(x,y,z) = (..,..,..)$

h. Give a composition of the two transformations
from problems 3 and 4 in the form
Comp(x,y,z) = (..,..)

5. Consider the linear transformation given by the standard matrix $A = \begin{bmatrix} 1 & 3 & 2 \\ 4 & -1 & 5 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

a. Give bases for the following.

domain of T

codomain of T

range of T

kernel of T

rank of T

nullity of T

b. Find the image of ((3,2,1).

c. Find the pre-image of (4,7,-2,3).

d. Find the pre-image of (1,1,1,1).

d. Write T as a rule.

e. Write T in terms of its effect on the standard basis.

f. Give reasons for your determination as to whether T is

one-to-one

onto

an isomorphism

g. If T is an isomorphism, write the inverse of T in the form $T^{-1}(x,y,z) = (..,..,..)$.

6. Consider the linear transformation T(x,y,z) = (x+y+z, x+y, y+z, x+y+z) given in terms of the implied standard bases S_3 for R^3 and S_4 for R^4. Use names and numbers for each of the following.

a. Suppose there is a nonstandard basis B in the domain R^3 where B = {(1,1,-1), (2,1,0), (3,1,0)}. Find the transformation matrix of T relative to B and S_4.

b. Now suppose there is a nonstandard basis E in R^4.
Let E = { (0.5,0,0,0), (0,1,0,1), (0,1,1,0), (0,0,1,1) }. Find the transformation matrix C of T relative to B and E.

c. Suppose a vector **v** in V has components (1,2,4) relative to the B basis. Find the components of T(**v**) relative to S_4.

6. (Continued) Now suppose there are other nonstandard bases D of R^3 and F of R^4 where D = { (1,1,3), (0,1,0) (2,1,7) } and F = { (1,0,1,0), (0,-1,2,0), (2,3,-5,0), (0,0,0,1) }.

d. Suppose a vector **v** in V has components (1,3,5) relative to the D basis. Find the components of T(**v**) relative to E.

e. Find the transition matrix P from D to B.

f. Find the transformation matrix G of T relative to D and F.

7. Consider the two vector spaces P_2 of polynomials of degree two or less and P_1 of polynomials of degree one or less.

Let { 1, x, x^2 } be the basis S for P_2.
Let { 1, x+3, $(x+3)^2$ } be the basis B for P_2.
Let { 1, x-5, $(x-5)^2$ } be the basis D for P_2.

Let { 1, x } be the basis U for P_1.
Let { 1, x+3 } be the basis E for P_1.
Let { 1, x-5 } be the basis F for P_1.

a. Write the bases B and D in terms of the basis S. One of the answers should look like

$\mathbf{b_2} = 3\ \mathbf{s_1} + 1\ \mathbf{s_2} = 3\ 1 + 1\ x$

b. Give the isomorphisms from P_2 to R^3. For **v** in P_2, one of these looks like

IsoB(**v**) = $[\mathbf{v}]_B$ meaning isoB($c_1+c_2(x+3)+c_3(x+3)^2$) = $\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$

IsoS

IsoD

c. Suppose a vector **v** in P_2 has the form $\mathbf{v} = 5 + 2(x+3) - 6(x+3)^2$. Find

$[\mathbf{v}]_B$

$[\mathbf{v}]_S$

$[\mathbf{v}]_D$.

7. (Continued)

d. Let T be the derivative transformation from P_2 to P_1, i.e., T(**v**) = the derivative of **v**.

So T(1) = 0, T(x) = 1 and $T(x^2) = 2x$. Give

$[T(1)]_U$,

$[T(x)]_U$

$[T(x^2)]_U$.

e. Give the standard matrix A for T (relative to S and U).

f. Give the matrix C for T relative to B and E.

g. For the vector **v** in part c, give $[T(\mathbf{v})]_E$