Name ____

1. Consider the matrix $A = \begin{bmatrix} 3 & -4 \\ -2 & 1 \end{bmatrix}$

   a. Find all real eigenvalues for the matrix A.

   b. Find the eigenvectors of A corresponding to the eigenvalues.

1. (continued)
   c. Give a basis for the eigenspace for one of the eigenvalues of A.

   d. Demonstrate the Cayley-Hamilton theorem for the matrix A.

   e. Find a matrix P (called the modal matrix in some texts) that demonstrates that A is diagonalizable.

   f. Use matrix multiplication to find A’s spectral matrix, the diagonal matrix to which A is diagonalizable.

2. Give a graphical description of eigenvalues and eigenvectors of a linear transformation in R^2.

3. Let A be a 3-by-3 symmetric matrix. Tell all you can about its eigenvalues, eigenvectors, diagonalizability and modal matrix P. (Hint: The Real Spectral Theorem and the Fundamental Theorem of symmetric matrices are a good place to start.)

4. For the symmetric matrix $A = \begin{bmatrix} 2 & 5 \\ 5 & 2 \end{bmatrix}$,

   a. find the eigenvalues and eigenvectors for A.
   b. give the spectral matrix for A.
   b. find an orthogonal modal matrix P for A.
   c. show that $P^T = P^{-1}$.
   d. show that P^TAP is the spectral matrix for A.

5. Give the 4X4 matrix for the homogeneous (linear) transformation which translates a coordinate frame (or an object lying in the coordinate frame) by an amount (2,3,4) from the base coordinate frame and then rotates the coordinate frame by 90° about the z axis of the base coordinate frame.

6. Tell what each of the columns of this homogeneous transformation matrix represents.

$$\begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 0 & 1 & -2 \\ 0 & -1 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

7. Give the product of four (Rot and Trans) transformation matrices that describe a link in a robotic manipulator arm. Tell what each variable or parameter represents as it relates to a link. (Do not multiply them together to form one matrix.)

8. For the particular revolute link on display in the front of the room, give the product of the four rotation/translation matrices for the A matrix of the link. Just use the names Rot and Trans with the correct variable and estimated values of the parameters to describe the links.