M 212 Lecture #1

While I'm taking roll, please recall some useful calculus.

1. For the function $y = f(x) = x^3 + 4x + 1$

a. Find the derivative $y' = f'(x)$, $= 3x^2 + 4$

b. find the equation of the tangent line @ $x = 2$.

$y - y_1 = m(x - x_1)$

$y - 17 = 16(x - 2)$

x	y	y'
2	17	16

2. Use the chain rule to find the derivative of

$$y = g(x) = (x^2 + 3x)^8$$

$$y' = g'(x) = 8(x^2 + 3x)^7 \cdot (2x + 3)$$

$\frac{dy}{dx}$

3. Use implicit differentiation to find y' in the equation

$$x^2 \cdot y^5 + \sin 2x + e^{3y} + \ln y = 10$$

$$x^2 5y^4 \cdot \frac{dy}{dx} + y^5 2x \cdot \frac{dx}{dx} + \cos(2x) \cdot 2\frac{dx}{dx} + e^{3y} \cdot 3\frac{dy}{dx} + \frac{1}{y} \cdot \frac{dy}{dx} = \frac{d0}{dx}$$

$$\left(5x^2 y^4 + 3e^{3y} + \frac{1}{y}\right) \frac{dy}{dx} = -2xy^5 - 2\cos 2x$$

$$\frac{dy}{dx} = \frac{-2xy^5 - 2\cos 2x}{5x^2y^4 + 3e^{3y} + \frac{1}{y}}$$

4. Use a substitution to integrate $\frac{1}{2}\int \sin^3(2x) \cdot \cos(2x)\, 2\, dx$

$u = \sin(2x)$

$du = \cos(2x) \cdot 2 \cdot dx$

$$= \frac{1}{2}\int u^3\, du$$

$$= \frac{1}{2} \frac{u^4}{4} + C$$

$$= \frac{u^4}{8} + C = \frac{\sin^4(2x)}{8} + C$$

M 212 Lect #1 1-20-10

Differential Equations (DE) in a Nut Shell

Activity	Calculus	DE
Begin with	$y = f(x) = 2x^2 - 11$	$\underbrace{\underset{DE}{y' = 4x}, \ \& \ \underset{IC}{y(3) = 7}}_{IVP}$ initial value problem
Evaluate the fcn	$f(3) = 7$	
Diff	$y' = f'(x) = 4x$	
Eval derivatives	$f'(3) = 12$	
	$f''(x) = 4$	$y'' = 4$
	$f''(3) = 4$	

Calculus:

x	y	y'	y''
3	7	12	4
4	21		

DE:

x	y	y'	y''
3	7	12	4
4	?	16	4

Solve IVP

$$y = \int y'\,dx = \int 4x\,dx = 2x^2 + C$$

Apply IC ($y = 7$, $x = 3$)

$$7 = 2 \cdot 3^2 + C$$
$$7 = 18 + C$$
$$-11 = C$$

Soln to the IVP is

$$\boxed{y = 2x^2 - 11}$$

We first will solve an IVP with a method called <u>Separation of Variables</u> (SOV)

$$\frac{(3x^3+1)\sec y\,dx}{x\,\sec y} = \frac{x\,dy}{x\,\sec y}, \quad y(1) = \frac{\pi}{6}$$

$$\underbrace{(3x^3+1)\sec y}\,dx \;\underbrace{-\,x}\,dy = 0$$

$$M(x,y)\,dx + N(x,y)\,dy = 0$$

$$\int \frac{3x^3+1}{x}\,dx = \int \cos y\,dy \qquad \text{Vars are Sep'd}$$

$$\int\left(3x^2 + \frac{1}{x}\right)dx = \sin y + C$$

$$x^3 + \ln|x| = \sin y + C$$

"general soln"

one parameter family of solns

implicit solution

Apply IC $y(1) = \frac{\pi}{6}$

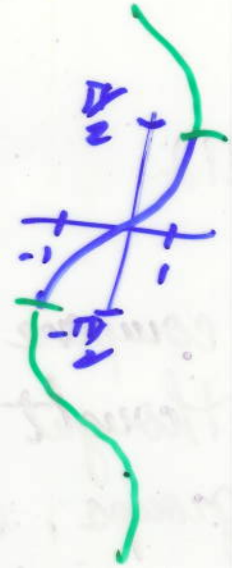
$$1^3 + \overset{0}{\cancel{\ln|1|}} = \sin\frac{\pi}{6} + C$$

$$1 + 0 = \frac{1}{2} + C$$

$C = \frac{1}{2}$. So the soln to the IVP is

$$\boxed{x^3 + \ln|x| = \sin y + \frac{1}{2}}$$

Let's seek an explicit soln.



$$\sin y = x^3 + \ln|x| - \tfrac{1}{2}$$

$$y = \sin^{-1}\left(x^3 + \ln|x| - \tfrac{1}{2}\right)$$

Over View of First Order IVP's

$$F(x, y, y') = 0 \ , \quad y(x_0) = y_0$$

First order in normal form

$$y' = f(x,y) \ = \ \frac{dy}{dx} = f(x,y) = -\frac{M(x,y)}{N(x,y)}$$

$$\frac{dy}{dx} = -\frac{M}{N}$$

$$-N\,dy = M\,dx$$

$$M(x,y)\,dx + N(x,y)\,dy = 0$$

Our Second Method for solving F.O. DE's

is called exact DE's.

In Calc III we do partial diff.

$$F(x,y) = x^2y^5 + e^{2x} + \ln y + 10$$

$$\frac{\partial F}{\partial x} = F_x = 2xy^5 + e^{2x}\cdot 2 + 0 + 0$$

Hold y fixed like a const

$$\frac{\partial F}{\partial y} = F_y = 5x^2y^4 + 0 + \frac{1}{y} + 0$$

$$M(x,y) = \sin(x\cdot y) + \cos x$$

$$M_y = \cos(x\cdot y)\cdot x + 0$$

$$N(x,y) = x^4y^6 + \tan x$$

$$N_x = 4x^3\cdot y^6 + \sec^2 x\cdot 1$$

$$dF = F_x\,dx + F_y\,dy$$

$$\begin{tikzcd}
 & F(x,y) \arrow[dl] \arrow[dr] & \\
M = F_x \arrow[dr] & & F_y = N \arrow[dl] \\
 & M_y = F_{xy} = F_{yx} = N_x &
\end{tikzcd}$$

For the DE

$M(x,y)\,dx + N(x,y)\,dy = 0$

① If $M_y = N_x$

② then $F(x,y) = C$

is the general soln

<u>Ex</u> $(3x^2y^5 + 2e^{2x})dx + (5x^3y^4 + \cos y)\,dy = 0$

$M(x,y)$ $\qquad\qquad$ $N(x,y)$

① The Test for Exactness "Bottom of Diamond"

$$M_y = 15x^2y^4 + 0 \equiv N_x = 15x^2y^4 + 0$$

Hence DE is exact!

② $F(x,y) = \int M\,dx = \int (3x^2y^5 + 2e^{2x})dx = x^3y^5 + e^{2x} + g(y)$ ← arb fcn

$F(x,y) = \int N\,dy = \int (5x^3y^4 + \cos y)dy = x^3y^5 + \sin y + h(x)$

$F(x,y) = x^3y^5 + e^{2x} + \sin y$

$$\boxed{x^3y^5 + e^{2x} + \sin y = C}$$

is the one param fam of solns