M212 Lect #3 1-27-10

Let's see how the IF (of one variable) $u(x)$ came about.

Begin with $M(x,y)\,dx + N(x,y)\,dy = 0$ DE

Assume that an IF $u = u(x)$ exist.

What will it look like

$$\underbrace{uM}_{m}\,dx + \underbrace{u\cdot N}_{n}\,dy = 0$$ must be exact.

The $m_y = n_x$

$$u M_y = u N_x + u_x N = u N_x + N u'$$

$$u M_y - u N_x = N u'$$

$$u \frac{M_y - N_x}{N} = u'$$

So also a fcn of x

fcn of x alone by hyp

$$\int \frac{M_y - N_x}{N} = \int \frac{u'}{u}$$

$$\int \frac{M_y - N_x}{N} dx = \ln u$$

$$e \qquad e$$

$$e^{\int \frac{M_y - N_x}{N}} = u$$

So if $u(x)$ is an IF, it has to look like

$$u = e^{\int \frac{M_y - N_x}{N} dx}$$

Clairaut's DE

$$y = y'x + (y')^2 \quad \Big| \quad y = y'x + f(y')$$

$$f(y') = (y')^2$$

$$f(t) = t^2 \qquad f'(t) = 2t$$

The one param fam of solns is

$$\boxed{y = cx + c^2}$$ looks like $y = mx + b$

$$y = cx + f(c)$$

Let's test:

$y' = c$ plug into DE

$$cx + c^2 \overset{\checkmark}{=} cx + c^2 \ \checkmark$$

Here is the <u>singular</u> soln

$$x = -2t \Rightarrow t = \frac{x}{-2}$$

$$y = t^2 - t(2t) = -t^2$$

$$x = 0f(t) - 1f'(t)$$

$$y = 1f(t) - tf'(t)$$

Eliminate the t

$$y = -\left(\frac{x}{-2}\right)^2 = -\frac{x^2}{4}$$

By chain rule

$$y' = t$$

Let's graph sing soln, then one param fam of solns

$y = -\frac{x^2}{4}$

x	y
± 2	-1
± 4	-4

$y = cx + c^2$

$y = 0$	$c=0$
$y = x+1$	$c=1$
$y = -x+1$	$c=-1$
$y = 2x+4$	$c=2$