

M212

Lect #3

(-27-10)

Let's see how the IF (of one variable) $u(x)$ came about.

Begin with $M(x,y) dx + N(x,y) dy = 0$ DE

Assume that an IF $u = u(x)$ exist.

What will it look like

$$\underbrace{u M dx}_M + \underbrace{u \cdot N dy}_N = 0 \text{ must be exact.}$$

$$\text{The } M_y = N_x$$

$$u M_y = u N_x + u_x N = u N_x + N u'$$

$$u M_y - u N_x = N u'$$

$$u \frac{M_y - N_x}{N} = u'$$

So also a fun of x
fun of x alone by hyp

$$\int \frac{M_y - N_x}{N} = \int \frac{u'}{u}$$

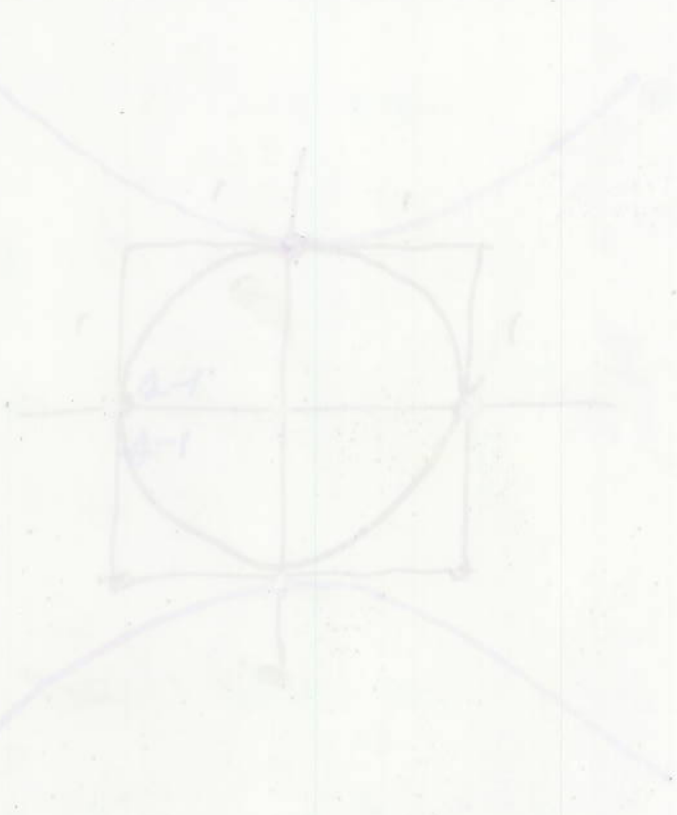
$$\int \frac{M_y - N_x}{N} dx = \ln u$$

The general form is

$$\int \frac{M_y - N_x}{N} = u$$

So if $u(x)$ is an IF, it has to look like

$$u = \int \frac{M_y - N_x}{N} dx$$



$x^2 + y^2 = 1$
 $-x^2 - y^2 = 1$
 no graph

I showed surfaces, miss at this point.

Clairaut's DE

$$y = y'x + (y')^2 \quad | \quad y = y'x + f(y')$$

$$f(y') = (y')^2$$

$$f(t) = t^2$$

$$f'(t) = 2t$$

The one param fam of solns is

$$y = cx + c^2$$

looks like $y = mx + b$

$$y = cx + f(c)$$

Let's test:

$$y' = c \quad \text{plug into DE}$$

$$cx + c^2 \stackrel{\checkmark}{=} cx + c^2 \checkmark$$

Here is the singular soln

$$x = -2t \Rightarrow t = \frac{x}{-2}$$

$$y = t^2 - t(2t) = -t^2$$

$$x = 0 \cdot f(t) - 1 \cdot f'(t)$$

$$y = 1 \cdot f(t) - t \cdot f'(t)$$

Eliminate the t

$$y = -\left(\frac{x}{-2}\right)^2 = -\frac{x^2}{4}$$

By chain rule

$$y' = t$$

Let's graph sing soln, then one param fam of solns

$$y = -\frac{x^2}{4}$$

x	y
±2	-1
±4	-4

$$y = 4x + C^2$$

$$y = 0$$

$$y = x + 1$$

$$y = -x + 1$$

$$y = 2x + 4$$

$$C=0$$

$$C=1$$

$$C=-1$$

$$C=2$$

