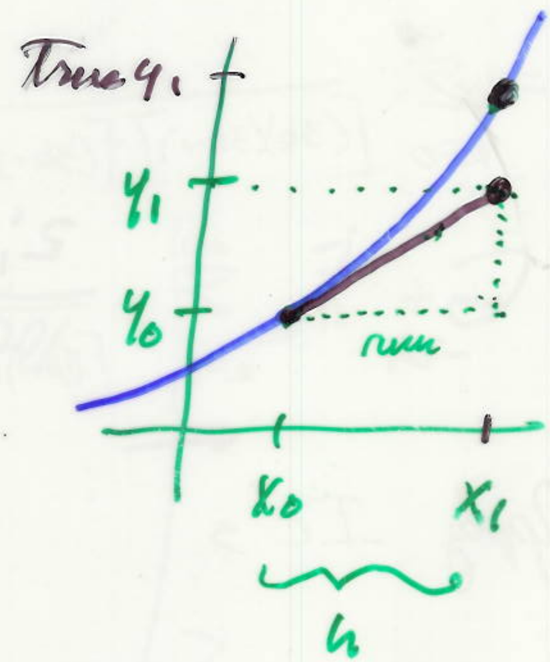
M 212 Lect #4 2-1-10

Let's now do a numerical method called the Euler Method to get a numerical solution to a IVP on an interval $[a,b]$

$$y' = f(x,y), \quad y(\overset{a}{x_0}) = y_0$$



$\left\{\begin{array}{l} x_0 = \text{given} \\ y_0 = \text{given} \end{array}\right\}$ as IC

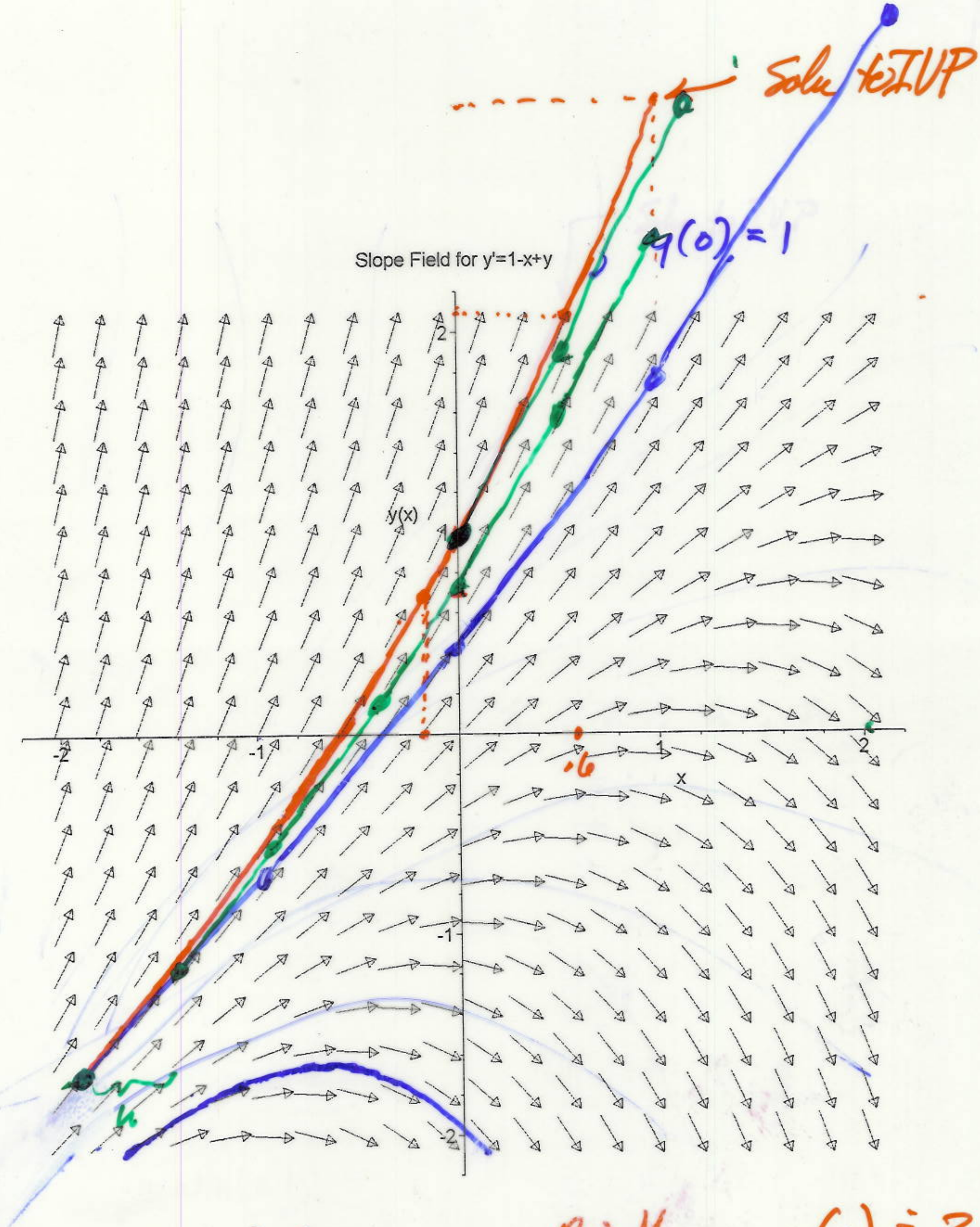
$\left\{\begin{array}{l} x_1 = x_0 + \text{run} = x_0 + h \\ y_1 = y_0 + \text{rise of tan line} \end{array}\right.$

$= y_0 + \text{run of TL} \cdot \dfrac{\text{rise of TL}}{\text{run of TL}}$ $\qquad h = \dfrac{b-a}{n}$

$= y_0 + \text{run} \cdot \text{slope}$

$= y_0 + h \cdot \text{deriv @ } x_0 \quad = y_0 + h \cdot y' \text{ @ } x_0$

$= y_0 + h \cdot f(x_0, y_0)$



Guess $y(.6) \doteq 2.1$

$y(-.2) \doteq .7$

Big Guess $y(1) \doteq 3$

$y' = f(x,y)\ ,\ \ y(x_0) = y_0$

$$\begin{cases} x_0 = a = \text{given} \\ y_0 = y(x_0) = \text{given} \end{cases}$$

$$\begin{cases} x_1 = x_0 + h \\ y_1 = y_0 + h \cdot f(x_0, y_0) \end{cases}$$

$$x_2 = x_1 + h$$
$$y_2 = y_1 + h\,f(x_1, y_1)$$

$$x_3 = x_2 + h$$
$$y_3 = y_2 + h\,f(x_2, y_2)$$

$\vdots$

$$x_{n+1} = x_n + h$$
$$y_{n+1} = y_n + h\,f(x_n, y_n)$$

Num Soln to IVP is

$$\boxed{\{(0,1), (.5,2), (1,3.25), (1.5, 4.875), (2, 7.0625)\}}$$

$y' = 1 - x + y\ ,\ y(0) = 1$ p3

on $[0,2]$ $n = 4$ iterations

$$\begin{cases} x_0 = 0 \\ y_0 = 1 \end{cases} \qquad h = \frac{2-0}{4} = \frac{b-a}{n}$$

$x_1 = x_0 + h = 0 + .5 = .5$

$y_1 = y_0 + h(1 - x_0 + y_0)$

$= 1 + (.5)(1 - 0 + 1) = 1 + 1 = 2$

$x_2 = x_1 + h = .5 + .5 = 1$

$y_2 = y_1 + h(1 - x_1 + y_1)$

$= 2 + (.5)(1 - .5 + 2) = 3.25$

$x_3 = x_2 + h = 1 + .5 = 1.5$

$y_3 = y_2 + h(1 - x_2 + y_2)$

$= 3.25 + (.5)(1 - 1 + 3.25)$

$= 3.25 + 1.625 = 4.825$

$x_4 = x_3 + h = 1.5 + .5 = 2$

$y_4 = y_3 + h(1 - x_3 + y_3)$

$= 4.875 + (.5)(1 - 1.5 + 4.875)$

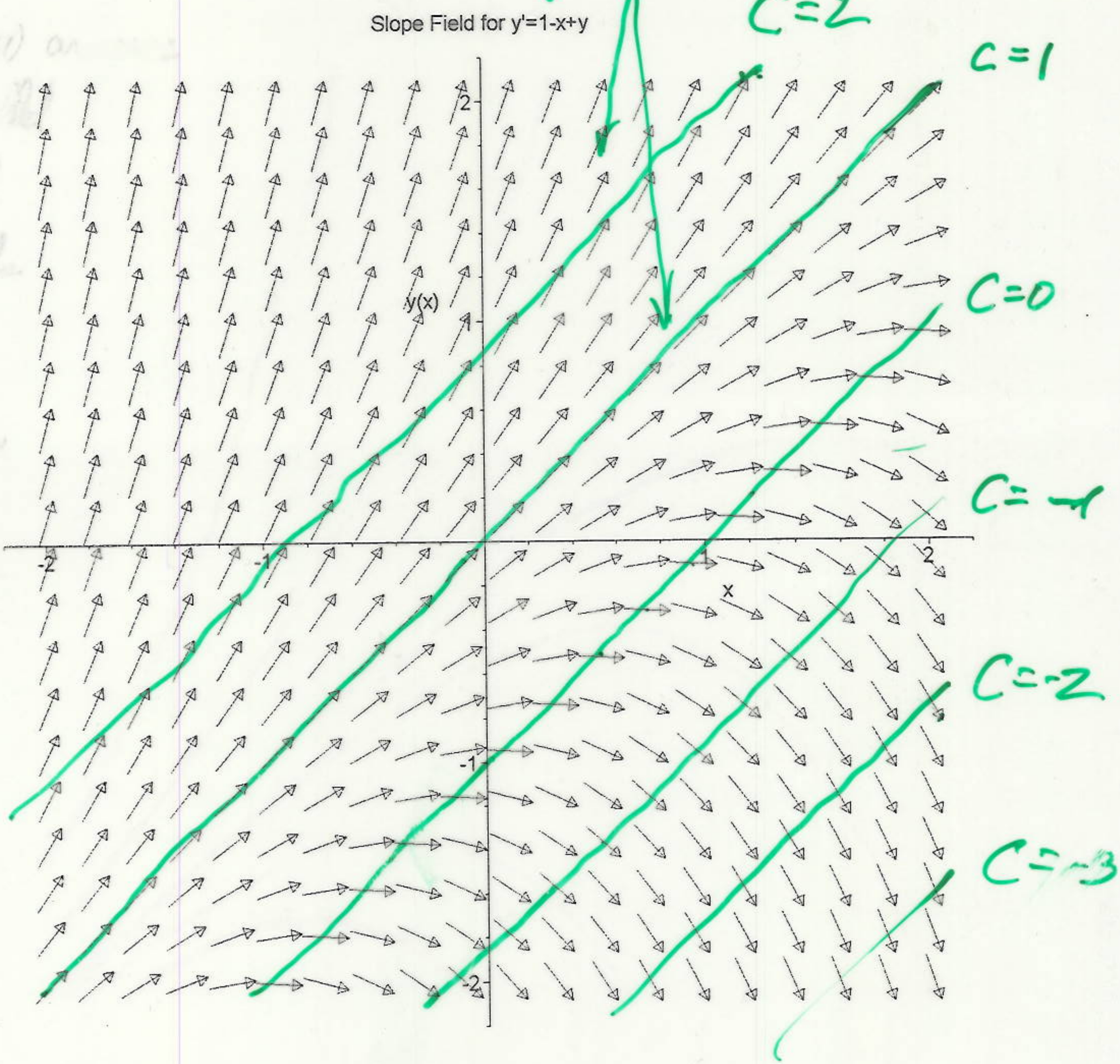
$= 4.875 + (.5)(4.375)$

$= 4.875 + 2.1825$

$= 7.0625$

P4

Isoclines for the DE



How do we get the eqns for the isoclines for this DE $y' = 1 - x + y$

Set $y' = C = 0$

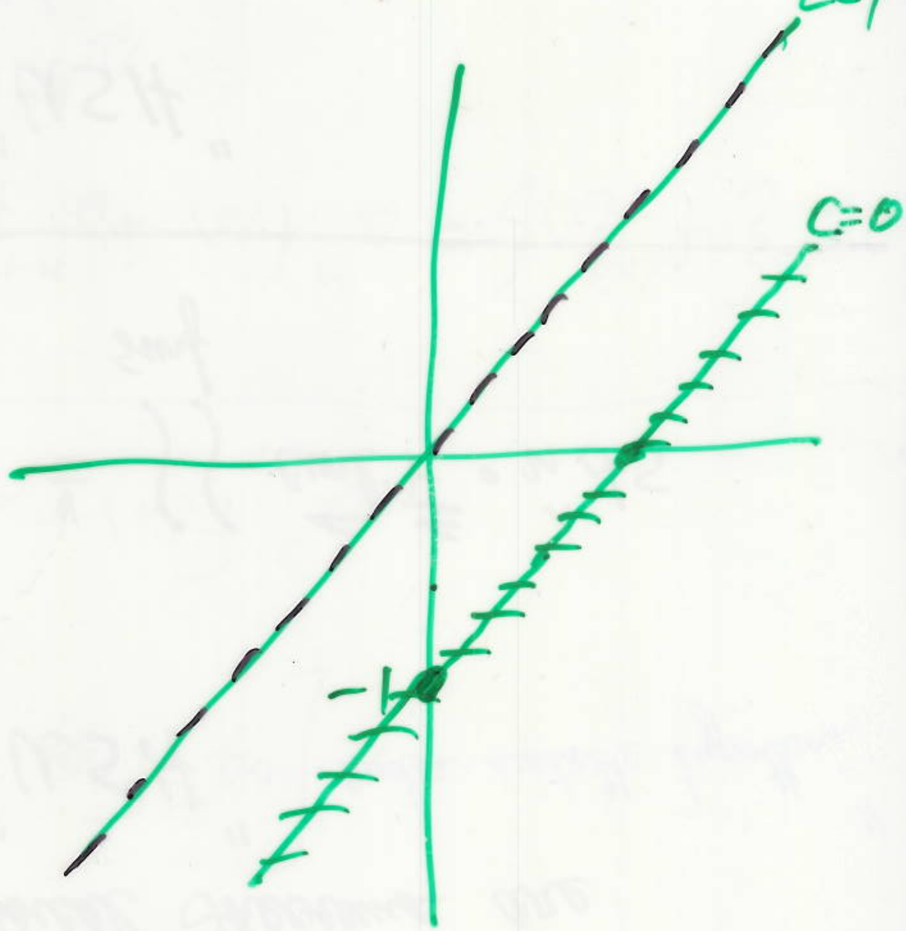
$$0 = 1 - x + y$$

$$y = 1x - 1$$

$$y' \overset{\text{set}}{=} C = 1$$

$$1 = 1 - x + y$$

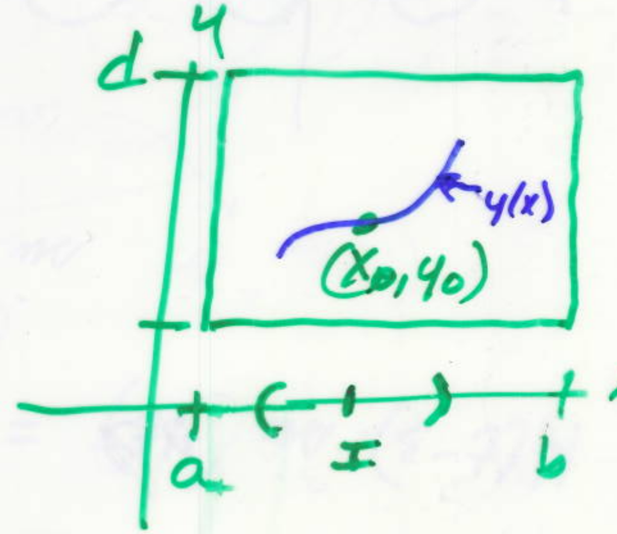
$$y = x$$



Picard's Theorem for F.O. IVP's

Consider the IVP $y' = f(x,y)$, $y(x_0) = y_0$

Let R be a rectangle



$$\left.\begin{array}{l} a \le x \le b \\ c \le y \le d \end{array}\right\} \text{ with } (x_0,y_0) \text{ in the interior}$$

① If $f(x,y)$ and $f_y(x,y)$ are both cont on the rectangle R.

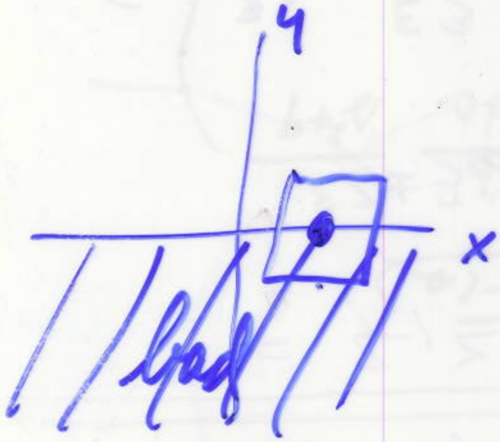
Then there exist an interval I with x_0 in its interior (or center) and (there exists) a unique $y(x)$ for which is the solution of the IVP on the interval I.

Ex See whether Picard's Th guarantees a unique soln on some interval about

a) $(1,0)$ b) $(2,3)$

for the IVP $y' = x y^{1/2}$

a) $y(1) = 0$



$f(x,y) = x y^{1/2}$

$f_y(x,y) = x \tfrac{1}{2} y^{-1/2}$

$= \dfrac{x}{2\sqrt{y}}$

Hyp is not met.
No conclusion can be drawn

b) $y(2) = 3$

R: $-4 \le x \le 6$
$1 \le y \le 5$

$(2,3)$ is inside

f + f_y are cont in R

So hyp is met

So there exists and interval I about $x_0 = 2$ such that the IVP has a unique soln in the interval I

Homogeneous DE's

$$(x^2 - xy)\,dx + \left(\frac{y^3}{x} + y^2\right)dy = 0$$

is homog of degree 2

If DE is homog, this subst
always converts DE to a SOV problem

$$y = ux \qquad dy = u\,dx + x\,du$$