

M. 212

Lect #5

2-9-10

Orthogonal Trajectories

OT

math, num

$$y = x^2 + C_1$$



-6, -4, -2, 0

$$y = -\frac{\ln|x|}{2} + C_2$$



-1, 0, 1

Shown on TI

$$\left\{ \begin{array}{l} y = x^2 + C_1 \\ y' = 2x \end{array} \right\} \text{ given fam}$$

← eliminate the arb const C_1

So $y' = 2x$ is the DE of the given family

$y' = -\frac{1}{2x}$ is the DE of the OT of given family

$$\frac{dy}{dx} = -\frac{1}{2x}$$

$$dy = -\frac{1}{2x} dx$$

Var are Sep'd

$$\int dy = -\frac{1}{2} \int \frac{dx}{x}$$

$$y = -\frac{1}{2} \ln|x| + C$$

is the Orthog Traj of the given family.

Higher Order Linear DE's

p2

FOL was $a(x)y' + b(x)y = c(x)$

Second OL $a_2(x)y'' + a_1(x)y' + a_0(x)y = b(x)$

n^{th} order linear

$$a_n(x)y^{(n)} + \dots + a_0(x)y = b(x),$$

$$y(x_0) = y_0, \dots, y^{(n-1)}(x_0) = y_{n-1}$$

The first Higher order Linear DE we do are call linear homogenous constant coeff

LHCC

Ex $2y'' - 13y' - 7y = 0$

aux eq: $2m^2 - 13m - 7 = 0$

$$(2m+1)(m-7) = 0$$

$$2m+1=0$$

$$m-7=0$$

$$m = -\frac{1}{2}$$

$$m = 7$$

The gen'l soln, i.e. the two parameter' fam of solns is

$$y = C_1 e^{-\frac{1}{2}x} + C_2 e^{7x}$$