M 212 Lect #6 2-15-10

Solving LHCC

① Distinct real roots case

$$y'' - 3y' - 4y = 0$$

Aux eq

$$m^2 - 3m - 4 = 0$$

$$(m+1)(m-4)$$

$$m = -1, 4$$

General Soln is

$$y = c_1 e^{-x} + c_2 e^{4x}$$

Check Soln: $y = c_1 e^{-x} + c_2 e^{4x}$

$$y' = -c_1 e^{-x} + 4c_2 e^{4x}$$

$$y'' = c_1 e^{-x} + 16 c_2 e^{4x}$$

Plug them in:

$$c_1 e^{-x} + 16 c_2 e^{4x} + 3c_1 e^{-x} - 12 c_2 e^{4x} - 4c_1 e^{-x} - 4 c_2 e^{4x}$$

$$\equiv 0 \checkmark$$

Case ② Repeated real roots

$$y'' - 10y' + 25y = 0$$

$$m^2 - 10m + 25 = 0$$

$$(m-5)(m-5)$$

$$m = 5, 5$$

We are tempted to

$$y = c_1 e^{5x} + c_2 e^{5x}$$

or

$$y = c_3 e^{5x}$$ ← This is not a 2param fam of solns, thus not the general soln.

We do get the general soln by mpying one of the term by x.

So the gen'l soln is

$$y = c_1 e^{5x} + c_2 x e^{5x}$$

Suppose roots were $m = 6, 6, 6$

$$y = c_1 e^{6x} + c_2 x e^{6x} + c_3 x^2 e^{6x}$$

Case (3) Complex Roots (Distinct)

$$e^{i\theta} = \cos(\theta) + i\sin(\theta)$$

Ex $y'' - 4y' + 13y = 0$

$$m^2 - 4m + 13 = 0$$

$$m = \frac{-(-4) \pm \sqrt{(-4)^2 - 4\cdot 1\cdot 13}}{2\cdot 1}$$

$$= \frac{4 \pm \sqrt{16-52}}{2} = \frac{4 \pm \sqrt{-36}}{2}$$

$$= \frac{4 \pm 6i}{2} = \frac{\cancel{2}(2 \pm 3i)}{\cancel{2}} = 2 \pm 3i$$

$$y = c_3 e^{(2+3i)x} + c_4 e^{(2-3i)x}$$

$$= c_3 e^{2x} \cdot e^{3xi} + c_4 e^{2x} e^{(-3x)i}$$

$$= e^{2x}\left[c_3 e^{(3x)i} + c_4 e^{(-3x)i}\right] \cdots$$

$$y = e^{2x}\left[c_1 \cos(3x) + c_2 \sin(3x)\right]$$

Suppose we have these roots

$m = 5 \pm 2i, \ 5 \pm 2i$. Genl Soln is

$$y = e^{5x}\left[C_1 \cos(2x) + C_2 \sin(2x) + C_3 x \cos(2x) + C_4 x \sin(2x)\right]$$