M. 212 Lect #8 2-22-10

One more piece for UC in solving LNH

Ex $y'' - 3y' - 4y = 42e^{4x}$

$m = 4, -1$

$y_c = c_1 e^{4x} + c_2 e^{-x}$

$y_t = a e^{4x}$ Uh, there is a duplicate in y_c

Bump up y_t by a factor of x

New $y_t = a x e^{4x}$

What if $y'' - 3y' - 4y = 42x^2 e^{4x} + \cos(2x)$

$y_t = a x^2 e^{4x} + b x e^{4x} + c e^{4x} + d \cos(2x) + e \sin(2x)$

aha, a dup in y_c

New $y_t = a x^3 e^{4x} + b x^2 e^{4x} + c x e^{4x}$

No more dups

B: amt of bacteria grows at a rate proportional to the amount present at any time

$$\frac{dB}{dt} = k \cdot B \quad , \quad B(0) = 4 \quad , \quad B(5) = 32$$

$$\frac{dB}{B} = k\,dt$$

Var are Sep'd

$$\ln|B| = kt + C$$

4 0

$$\ln 4 = C$$

$$\ln B = kt + \ln 4$$

$$\ln \frac{B}{4} = kt$$

$$\ln\left(\frac{32}{4}\right) = k \cdot 5$$

$$k = \frac{\ln 8}{5} \approx .415$$

Soln is

$$\boxed{\ln \frac{B}{4} = .415t}$$

Annihilators - A Diff operator that makes fcns disappear

To Solve LH:

$f(D)y = 0$ LH

$(D^2-3D-4)y = 0$

$m^2-3m-4 = 0$ AE

$(m-4)(m+1) = 0$

$m = 4, -1$

$y_c = c_1e^{4x} + c_2e^{-x}$

To find annihilator for e^{4x} and e^{-x}

$m = 4, -1$

$(m-4)(m+1) = 0$

$m^2-3m-4 = 0$

$f(D) = D^2-3D-4$

is the annihilator for e^{4x} and e^{-x}

Also

$f(D) = (D-4)(D+1)$

Find annih for

$x^2 e^{13x}$

$m = 13, 13, 13$

$(m-13)^3 = 0$ AE

.....

$f(D) = (D-13)^3$

is ann. for

$e^{6x} \cdot (\cos(5x) + 0 \sin(5x))$

$m = 6 \pm 5i$

$(m - (6+5i)) \cdot (m - (6-5i))$

$((m-6) - 5i) \cdot ((m-6) + 5i)$

$(m-6)^2 - 25i^2$

$(m-6)^2 + 25$

$m^2 - 12m + 61 = 0$ AE

$i^2 = \sqrt{-1}^2$

$i^2 = -1$

ann is $D^2 - 12D + 61$

Solve this DE (LH)

$$(2D^3 - 13D^2 + 24D - 9)\, y = 0$$

$$2m^3 - 13m^2 + 24m - 9 = 0$$

$\uparrow$ P $\qquad\qquad$ $\uparrow$ Q

$p = 1, 2$ $\qquad\qquad$ $q = 1, 3, 9$

$$\pm \frac{q}{p} = \pm \frac{1, 3, 9}{1, 2}$$

Any rat root of the eq above

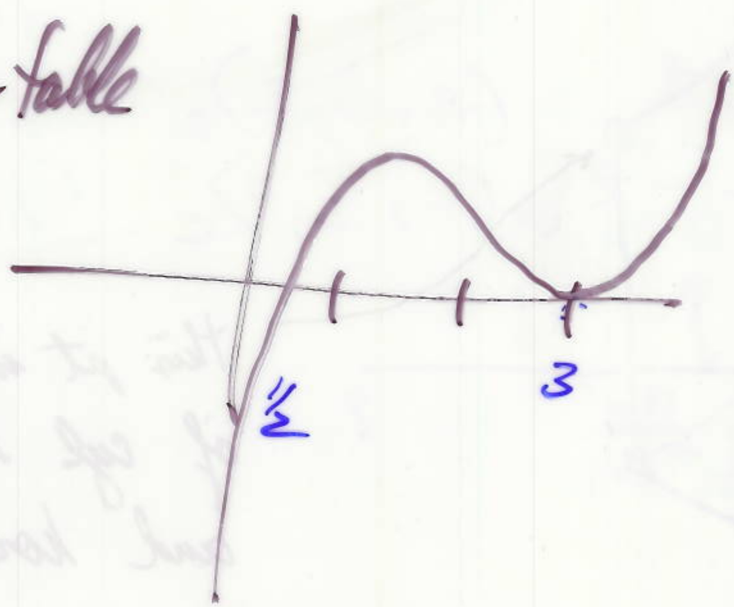
is of the form $\pm \frac{q}{p} = \pm\frac{1}{1}, \pm\frac{3}{1}, \pm\frac{9}{1}, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{9}{2}$

From the graph & table

we conclude

$m = \frac{1}{2}, 3, 3$

are the roots



P6

Solve this LNH

$$y'' - 3y' - 4y = 42e^{6x}$$ by Variation of Parameters VOP

① Solve LH

$m^2 - 3m - 4 = 0$

$m = 4, -1$

$y_c = C_1 e^{4x} + C_2 e^{-x}$

$y_t = A(x)\, e^{4x} + B(x)\, e^{-x}$

$y_t' = A(x) e^{4x} \cdot 4 + A'(x) e^{4x} + (-1) B(x) e^{-x} + B'(x) e^{-x}$

$= 4A(x) e^{4x} - B(x) e^{-x} + \underbrace{A'(x) e^{4x} + B'(x) e^{-x}}_{\text{Cond ① is to set this to 0}}$

$y_t'' = 16 A(x) e^{4x} + 4 A'(x) e^{4x} + B(x) e^{-x} - B'(x) e^{-x}$

Cond ② is that y_t work in LNH

Plug into LNH

$$16A(x)e^{4x} + 4A'(x)e^{4x} + B(x)e^{-x} - B'(x)e^{-x}$$
$$-12A(x)e^{4x} \qquad +3B(x)e^{-x}$$
$$-4A(x)e^{4x} \qquad -4B(x)e^{-x} = 42e^{6x}$$

$$\begin{cases} 4A'(x)e^{4x} - B'(x)e^{-x} = 42e^{6x} & (2) \\ A'(x)e^{4x} + B'(x)e^{-x} = 0 & (1) \end{cases}$$

$$\begin{cases} A'(x)e^{4x} + B'(x)e^{-x} = 0 \\ A'(x)4e^{4x} + B'(x)(-1)e^{-x} = RHS = 42e^{6x} \end{cases}$$

This time add eq

$$5e^{4x}A'(x) = 42e^{6x}$$

$$A'(x) = \frac{1}{2}\frac{42}{5}e^{2x}\,2\,dx$$

$$A(x) = \frac{21}{5}e^{2x} + C_1$$

Similarly Solve for $B(x)$

$\vdots$

Then Plug $A(x)$ & $B(x)$ into y_t

y_p or y_g = answer