M212 Lect # 11 3-3-10

We solve a small system of DE's

$$\begin{Bmatrix} \dot{x} = 8y \\ \dot{y} = 2x \end{Bmatrix}$$

a large example might have $\frac{dx}{dt} = 2x + 3y + e^t$

$$\begin{Bmatrix} Dx - 8y = 0 \\ -2x + Dy = 0 \end{Bmatrix}$$

$(D-2)x + (D+3)y = e^{2t}$

$$x = \frac{\begin{vmatrix} 0 & -8 \\ 0 & D \end{vmatrix}}{\begin{vmatrix} D & -8 \\ -2 & D \end{vmatrix}}.$$

$\frac{20}{4} = 5$ means $20 = 5 \cdot 4$

$$\begin{vmatrix} D & -8 \\ -2 & D \end{vmatrix} x = \begin{vmatrix} 0 & -8 \\ 0 & D \end{vmatrix}$$

$$(D^2 - 16)x = D \cdot 0 + 8 \cdot 0$$

$$(D^2 - 16)x = 0$$

$m^2 - 16 = 0$ $\quad m = \pm 4$

$$2x + 3y = 8$$
$$4x + 5y = 9$$

Cramer's Rule

$$x = \frac{\begin{vmatrix} 8 & 3 \\ 9 & 5 \end{vmatrix}}{\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix}} \qquad y = \frac{\begin{vmatrix} 2 & 8 \\ 4 & 9 \end{vmatrix}}{\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix}}$$

$$y = \frac{40 - 27}{2\cdot 5 - 4\cdot 3} \qquad y = \frac{18 - 32}{10 - 12}$$

$$x = \frac{13}{-2} \qquad y = \frac{-14}{-2} = 7$$

$x_c = x_g = c_1 e^{-4t} + c_2 e^{4t}$

This is gen'l soln for the 2nd order LH

Now we seek y_g. Solve Eq ① for

y: $\quad \dot{x} = 8y \qquad y = \frac{1}{8}\dot{x}$

Plug x_g into this

$y_g = \frac{1}{8}\left((-4)c_1 e^{-4t} + 4c_2 e^{4t}\right)$

$y_g = -\frac{1}{2}c_1 e^{-4t} + \frac{1}{2}c_2 e^{4t}$

So the general soln to the System of DE's is

$$\boxed{\begin{cases} x_g = c_1 e^{-4t} + c_2 e^{4t} \\ y_g = -\frac{1}{2}c_1 e^{-4t} + \frac{1}{2}c_2 e^{4t} \end{cases}}$$

Ready to plug in any IC's