M 212 Lect # 12 3-8-10

A Larger System of IVP's by Diff Operator Meth

$$\begin{cases} \dot{x} = 3x - y - 12 & (1) \\ \dot{y} = x + y + 4e^t & (2) \end{cases} \qquad \left.\begin{aligned} x(0) &= 0 \\ y(0) &= 1 \end{aligned}\right\} \text{IVP}$$

$$\begin{cases} (D-3)x + y = -12 \\ -x + (D-1)y = 4e^t \end{cases}$$

$$\begin{vmatrix} D-3 & 1 \\ -1 & D-1 \end{vmatrix} x = \begin{vmatrix} -12 & 1 \\ 4e^t & D-1 \end{vmatrix}$$

$$\left((D-3)(D-1)+1\right)x = (D-1)(-12) - 1 \cdot 4e^t$$

$$(D^2 - 4D + 3 + 1)x = 0 + 12 - 4e^t$$

$$(D^2 - 4D + 4)x = 12 - 4e^t \quad \text{LNHCC} \quad (3)$$

$m^2 - 4m + 4$

$(m-2)(m-2)$ $\quad m = 2, 2$

$$x_c = c_1 e^{2t} + c_2 t e^{2t}$$

Let's Use UC to get x_p, then x_g

$x_t = a\cdot 1 + be^t$ Compare with x_c No dups.

$\dot{x}_t = be^t\cdot 1 = Dx_t$ $\frac{d}{dt}e^{3t} = e^{3t}\cdot 3$

$\ddot{x}_t = be^t = D^2x_t$

$be^t - 4be^t + 4(a + be^t) = 12 - 4e^t$

$(b - \cancel{4b} + \cancel{4b})e^t + 4a\cdot 1 = -4e^t + 12\cdot 1$

$b = -4$ $a = \frac{12}{4} = 3$

$x_p = 3 - 4e^t$ so goal soln of 3

$x_g = x_c + x_p$

$$\boxed{x_g = c_1e^{2t} + c_2te^{2t} + 3 - 4e^t}$$

Officially

$(b+4)e^t + (4a-12)\cdot 1 = 0$

$c_1e^t + c_2\cdot 1 = 0$

$b + 4 \overset{set}{=} 0$ $4a - 12 \overset{set}{=} 0$

Let's solve for y in ① $\dot{x} = 3x - y - 12$

$y = 3x - \dot{x} - 12$ so

$y_g = 3x_g - \dot{x}_g - 12$ ⑥

Recall $x_g = c_1 e^{2t} + c_2 t e^{2t} + 3 - 4e^t$

Then $\dot{x}_g = 2c_1 e^{2t} + c_2 \, t \cdot 2e^{2t} + c_2 1 \, e^{2t}$

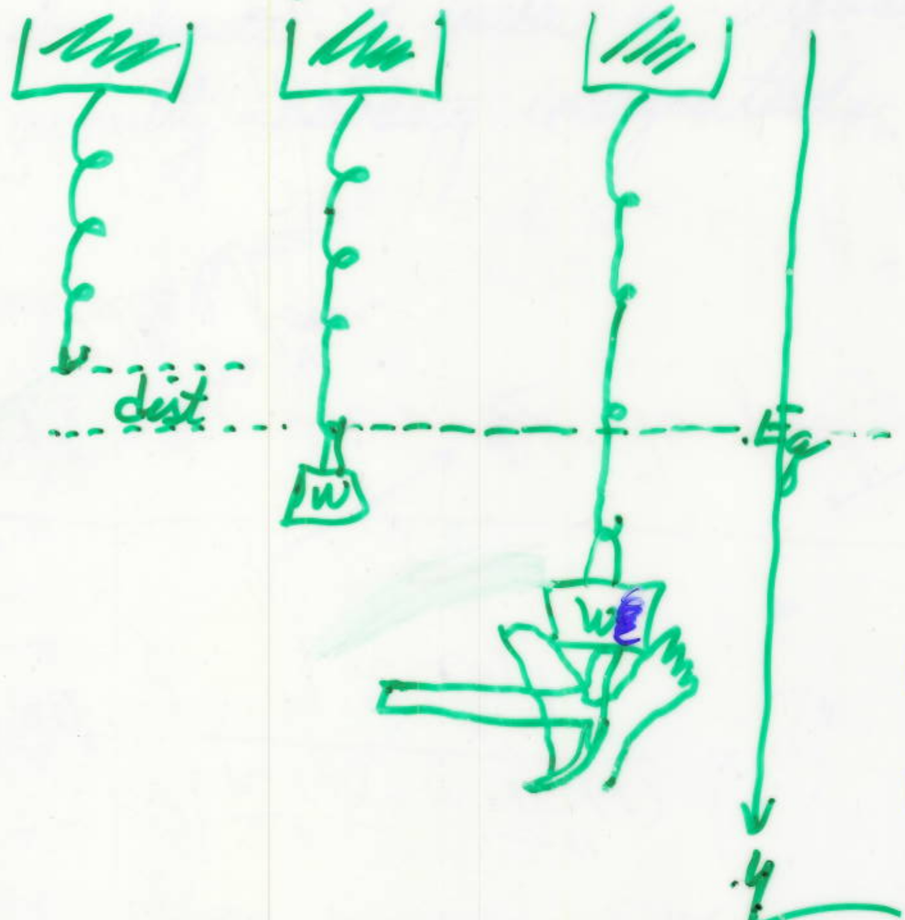
$+ 0 - 4e^t$

We plug these into ⑥

$$y_g = 3c_1 e^{2t} + 3c_2 t e^{2t} + 9 - 12e^t$$
$$-2c_1 e^{2t} - 2c_2 t e^{2t}$$
$$-c_2 e^{2t} \qquad -0 + 4e^t$$
$$-12$$

$$\boxed{y_g = \underbrace{(c_1 - c_2)}_{c_3} e^{2t} + \underbrace{c_2}_{c_4} t e^{2t} - 3 - 8e^t}$$

The spring problem



Newton's Second Law

$$F_T = m a$$

$$F_T = m y''$$

$$F = m g$$

$$m = \frac{F}{g}$$

external force

$$F_T = F_S + F_R + F_E(t)$$

Hooke's Law: $F_S = -ky$

proportional assumption $F_R = -By'$

$$-ky - By' + F_E(t) = my''$$

The DE for Spring Motion is

$$\boxed{my'' + By' + ky = F_E(t) \quad \begin{cases} y(0) = y_0 \\ y'(0) = v_0 \end{cases}}$$

$$my'' + \beta y' + ky = F_E(t),$$

$$y(0) = y_0, \quad y'(0) = v_0$$

$Wt = m \cdot g$ mass $= \boxed{m = \frac{w}{g}}$

β given in words : "$\boxed{\text{three}}$ times velocity"

k use $F = k\,dist \Rightarrow w = k\,dist$

$$\boxed{k = \frac{w}{dist}}$$