M 212 Lect # 13 3-10-10

Continuing with the large system

$x_g = c_1 e^{2t} + c_2 t e^{2t} + 3 - 4e^{t}$

$y_g = (c_1 - c_2)e^{2t} + c_2 t e^{2t} - 3 - 8e^{t}$

Apply IC's $x(0) = 0$ $y(0) = 1$

$0 = c_1 e^{0} + c_2 0 e^{0} + 3 - 4e^{0}$

$= c_1 + 0 + 3 - 4$

$0 = c_1 - 1$

$c_1 = 1$

$1 = (c_1 - c_2)e^{0} + \cancel{c_2 \cdot 0 \cdot e^{0}} - 3 - 8e^{0}$

$\cancel{1} = (\cancel{1} - c_2) - 3 - 8$

$0 = -c_2 - 11$ $c_2 = -11$

So soln to the IVP is

$$\boxed{\begin{aligned} x &= 1e^{2t} - 11te^{2t} + 3 - 4e^{t} \\ y &= 12e^{2t} - 11te^{2t} - 3 - 8e^{t} \end{aligned}}$$

Let' read the problem p 207 #5

mass weighs $20\# = W$

str sp $6'' = \frac{1}{2}$ ft

6'' below equil

below

$y(0) = +\frac{1}{2}$ ft

$y'(0) = 0 \frac{ft}{sec}$

from rest

Eq of motion is

$$my'' + \beta y' + ky = F_E(t)$$

0 no mention

$W = m \cdot g$

$m = \frac{W}{g} = \frac{20}{32}$

$m = \frac{5}{8}$

$B = 0$ no mention

$F = k \cdot$ dist

$20 = k \cdot \frac{1}{2}$

$k = 40$

IVP becomes

$$\boxed{\frac{8}{5}\,\frac{5}{8}y'' + \frac{8}{5}40y = 0\,\frac{8}{5}, \quad y(0) = \frac{1}{2}, \quad y'(0) = 0}$$

$y'' + 64y = 0$ ....

$y = c_1 \cos(8t) + c_2 \sin(8t)$

Apply IC's

What will the questions look like?

How far
When will
How fast

t	y	y'	y''

Solving DE's and IVP's by power series

Find a power series solution for this really easy DE

$$y'' + 16y = 0, \quad y(0) = 3 = a_0, \quad y'(0) = 0 = a_1$$

$$y_t = \sum_{n=0}^{\infty} a_n (x - x_0)^n$$

we often have $x_0 = 0$

$$y_t = \sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + \ldots$$

$$y''(0) = 4$$

$$y_t' = \sum_{\substack{n=0 \\ \text{or} \\ n=1}}^{\infty} n a_n x^{n-1} = 0 + a_1 + 2a_2 x + 3a_3 x^2 + 4a_4 x^3 + \ldots$$

$$y_t'' = \sum_{\substack{n=0 \\ \text{or } n=1 \\ \text{or } n=2}}^{\infty} n(n-1) a_n x^{n-2} = 0 + 0 + 2a_2 + 6a_3 x + 12a_4 x^2 + \ldots$$

Plug these into DE

$$\sum_{\substack{n=0 \\ \text{or } 1 \\ 2}}^{\infty} n(n-1) a_n x^{n-2} + 16 \sum_{n=0}^{\infty} a_n x^n = 0$$

$$\sum_{\substack{n=0 \\ 1 \\ 2}}^{\infty} n(n-1) a_n x^{n-2} + \sum_{n=0}^{\infty} 16 a_n x^n = 0$$

$$\sum_{\substack{n=0\\ \downarrow \\ 2}}^{\infty} n(n-1)a_n x^{n-2} + \sum_{n=0}^{\infty} 16 a_n x^n = 0$$

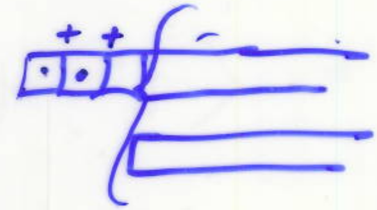
$$\sum^{\infty} (n+2)(n+2-1)a_{n+2} X^{n+2-2}$$

$n+2=0$
$n+2=1$
$n+2=2$

$$\sum^{\infty}(n+2)(n+1)a_{n+2}X^n + \sum_{n=0}^{\infty} 16 a_n X^n = 0$$

$n=-2$
or $n=-1$
or $n=0$

$\sum_{n=3}^{\infty}$



$$\sum_{n=0}^{\infty}(n+2)(n+1)a_{n+2}x^n + 16a_nx^n = 0$$

$$\sum_{n=0}^{\infty}\Big((n+2)(n+1)a_{n+2}+16a_n\Big)x^n = 0$$

Recall $\{1, x, x^2, x^3, x^4, \dots, x^n, \dots\}$ are LI

So $(n+2)(n+1)a_{n+2}+16a_n = 0$ for $n=0,1,2,\dots$

We solve for a_{n+2}

$$a_{n+2} = \frac{-16}{(n+1)(n+2)} a_n \qquad \text{for all } n = 0, 1, 2 \dots$$

<u>even ones</u>

$k=0 \quad a_0 = a_0 = 3$

$k=1 \quad n=0 \quad a_2 = \frac{-16}{1 \cdot 2} a_0$

$k=2 \quad n=2 \quad a_4 = \frac{-16}{3 \cdot 4} a_2 = \frac{-16}{4 \cdot 3} \frac{-16}{2 \cdot 1} a_0 = \frac{(-16)^2}{4!} a_0$

$k=3 \quad n=4 \quad a_6 = \frac{-16}{6 \cdot 5} a_4 = \frac{(-16)(-16)(-16)}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} a_0 = \frac{(-16)^3}{6!} a_0$

$\vdots$

$k=k \quad a_{2k} = \frac{(-16)^k}{(2k)!} a_0$

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odd one

$k=0 \quad a_1 = a_1 = 0$

$k=1 \quad n=1 \quad a_3 = \frac{-16}{2 \cdot 3} a_1$

$k=2 \quad n=3 \quad a_5 = \frac{-16}{4 \cdot 5} a_3 = \frac{(-16)(-16)}{5 \cdot 4 \cdot 3 \cdot 2} a_1 = \frac{(-16)^2}{5!} a_1$

$\vdots$

$k=k \quad a_{2k+1} = \frac{(-16)^k}{(2k+1)!} a_1$

$$y_t = \sum_{n=0}^{\infty} a_n x^n$$

$$= \sum_{k=0}^{\infty} a_{2k} x^{2k} + \sum_{k=0}^{\infty} a_{2k+1} x^{2k+1}$$

Power Series Solution

$$= \overset{3}{a_0} \sum_{k=0}^{\infty} \frac{(-16)^k}{(2k)!} x^{2k} + \overset{0}{a_1} \sum_{k=0}^{\infty} \frac{(-16)^k}{(2k+1)!} x^{2k+1}$$