M 212 Lect # 14 3-15-10

Like,
Airy's DE by power series

$y' - xy = 0$

$y_t = \sum_{n=0}^{\infty} a_n x^n$

$y_t' = \sum_{n=0}^{\infty} n a_n x^{n-1}$ $\qquad x^1 \cdot x^n$

$\sum_{n=0}^{\infty} n a_n x^{n-1} - x \sum_{n=0}^{\infty} a_n x^n = 0$

$\sum_{n=0}^{\infty} n a_n x^{n-1} - \sum_{n=0}^{\infty} a_n x^{n+1} = 0$

match form of x's expon.

$\sum_{n=-2} (n+2) a_{n+2} x^{n+1} - \sum_{n=0}^{\infty} a_n x^{n+1} = 0$

$n = -2$
$= -1 \leftarrow$

$+ 1 \cdot a_1 x^0 + \sum_{n=0}^{\infty} (n+2) a_{n+2} x^{n+1} - \sum_{n=0}^{\infty} a_n x^{n+1} = 0$

$a_1 + \sum_{n=0}^{\infty} \left((n+2) a_{n+2} - a_n \right) x^{n+1} = 0$

$a_1 = 0$ and $(n+2)a_{n+2} - a_n = 0$ for $n = 0, 1, 2 \dots$

$a_1 = 0$, $a_{n+2} = \dfrac{a_n}{n+2}$ $\quad n = 0, 1, 2$

k=0 $\quad a_0 = a_0$

k=1 $\quad$ n=0 $\quad a_2 = \dfrac{1}{2} a_0$

k=2 $\quad$ n=2 $\quad a_4 = \dfrac{1}{4} a_2 = \dfrac{1}{4 \cdot 2} a_0$

k=3 $\quad$ n=3 $\quad a_6 = \dfrac{1}{6} a_4 = \dfrac{1}{6 \cdot 4 \cdot 2} a_0$

k=k $\quad a_{2k} = \dfrac{1}{2 \cdot 4 \cdot 6 \cdots (2 \cdot k)} a_0$

$a_{2k} = \dfrac{}{2 \cdot 1 \cdot 2 \cdot 2 \;\; 2 \cdot 3}$

$a_{2k} = \dfrac{1}{2^k \, k!} a_0$

k=0 $\quad a_1 = a_1 = 0$ by method

k=1 $\quad$ n=1 $\quad a_3 = \dfrac{1}{3} a_1 = \dfrac{1}{3} \cdot 0 = 0$

k=2 $\quad$ n=3 $\quad a_5 = \dfrac{1}{5} a_3 = \dfrac{1}{5} \cdot 0 = 0$

k=k $\quad a_{2k+1} = \dfrac{\textcolor{red}{2 \cdot 4 \cdot 6 \cdot (2k)}\; 1}{\textcolor{red}{2}\, 3 \cdot \textcolor{red}{4}\, 5 \cdot \textcolor{red}{6}\, 7 \cdots \textcolor{red}{2k}\, (2k+1)} \cancel{a_1}^{\,0}$

$a_{2k+1} = \dfrac{2^k \, k!}{(2k+1)!} \cancel{a_1}^{\,0}$

$$y_t = \sum_{n=0}^{\infty} a_n x^n$$

$$= \sum_{k=0}^{\infty} a_{2k} x^{2k} + \sum_{k=0}^{\infty} a_{2k+1} x^{2k+1}$$

$$= a_0 \sum_{k=0}^{\infty} \frac{1}{2^k k!} x^{2k} + a_1 \sum_{k=0}^{\infty} \frac{2^k k!}{(2k+1)!} x^{2k+1}$$

0

$$\boxed{y = a_0 \sum_{k=0}^{\infty} \frac{1}{2^k k!} x^{2k}}$$

genl power series soln.

Now apply a newly discovered IC

$$y(0) = 0 = a_0$$

$$0 = a_0\left(\frac{1}{2^0 0!} 1\right)$$

$0 = a_0 \cdot 1$ so $a_0 = 0$

Pow Ser Soln to the IVP is $y = 0$

Convert ~~Change~~ this higher order IVP to a system of 1st order IVP's

$$y'''' + 2xy''' - 6e^x y'' - 3y' + 2y = \sin x$$

$y(0) = 6 \quad y'(0) = 7 \quad y''(0) = 8 \quad y'''(0) = 9$

$z_1(0) = 6 \quad z_2(0) = 7 \quad z_3 \quad z_4$

$$\underset{z_4'}{y''''} = -2x\underset{z_3'}{\overset{z_4}{y'''}} + 6e^x \underset{z_2'}{\overset{z_3}{y''}} + 3\underset{z_1'}{\overset{z_2}{y'}} - 2\overset{z_1}{y} + \sin x$$

$$\boxed{\begin{cases} z_1' = z_2 & z_1(0) = 6 \\ z_2' = z_3 & z_2(0) = 7 \\ z_3' = z_4 & z_3(0) = 8 \\ z_4' = -2x z_4 + 6e^x z_3 + 3z_2 - 2z_1 + \sin x, & z_4(0) = 9 \end{cases}}$$

So now we can convert
a system of 1st order IVP's to a higher order IVP

Just now we converted
a higher order IVP to a system of F.O. IVP's

Q. When do we need these two conversions.

A. When solving a system of IVP's algebraically
we first convert to a higher order.

When solve a higher order IVP numerically
we first convert to a system of F.O. IVP's