M212 Lect # 15 3-17-10

Recall the spring Motion IVP

$$my'' + by' + ky = F_E(t), \quad y(0) = y_0, \quad y'(0) = v_0$$

y is dep var

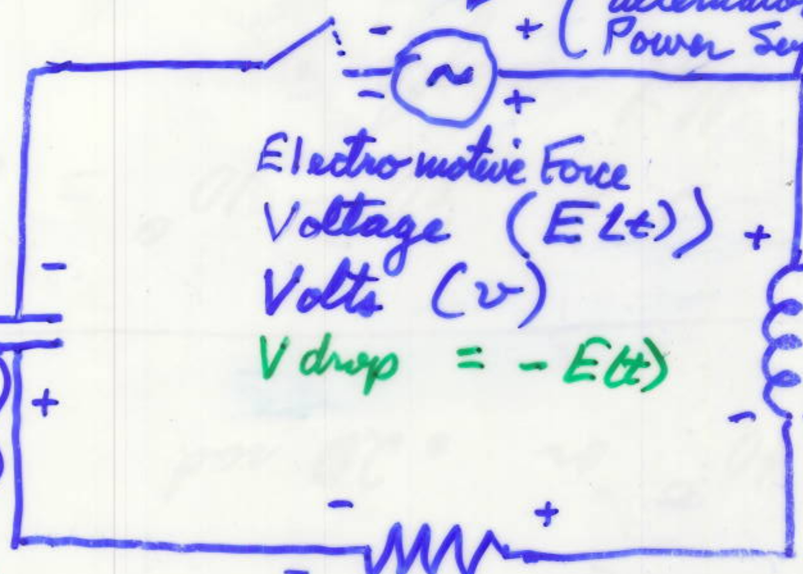
t is indep var

$$Lq'' + Rq' + \frac{1}{C}q = E(t), \quad q(0) = q_0, \quad q'(0) = i_0$$

indep var time (sec)

dep var, charge (coulombs)

battery
generator
alternator
Power Supply



Electromotive Force
Voltage $(E(t))$
Volts (v)
Vdrop $= -E(t)$

Capacitor
Capacitance (C)
Farads (f)
Vdrop $= \frac{1}{C}q$

Inductor
Inductance (L)
Henrys (h)
Vdrop $= Lq''$

Resistor
Resistance (R)
Ohms (Ω)
Vdrop $= Rq'$

Kirchoff 's Laws

1. The algebraic sum of the voltage drops around any closed loop in a circuit is zero, i.e. (the drops = the rises)

2. The currents entering any point in a circuit = the currents leaving that point

charge current

i

The questions

1. When will
2. How much
3. How fast
(How much current)

t	q	q'	q''

Chapter 7 Laplace Transforms

$$
\begin{tikzcd}
\text{IVP in } y \text{ and } t \arrow[r, "\text{solve}"] \arrow[d, "\mathcal{L}"] & \text{soln to IVP } y = y(t) \\
\text{Algebraic Eqn in } Y \text{ and } s \arrow[r, "\text{solve by algebra}"] & \text{soln to A.E. } Y = Y(s) \arrow[u, "\mathcal{L}^{-1}"]
\end{tikzcd}
$$

An Integral transform is a Operator

$$\mathcal{T}\{f(t)\} = \int_{-\infty}^{\infty} K(s,t)\, f(t)\, dt = F(s)$$

For Laplace Transform the kernel is

$$K(s,t) = \begin{cases} 0, & t < 0 \\ e^{-st}, & t \geq 0 \end{cases}$$

So

$$\mathcal{L}\{f(t)\} = \cancelto{0}{\int_{-\infty}^{0} 0 \cdot f(t)\, dt} + \int_{0}^{\infty} e^{-st} f(t)\, dt$$

1a)

$$\boxed{\mathcal{L}\{f(t)\} = \int_{0}^{\infty} e^{-st} f(t)\, dt \quad \text{provided the improper integ conv for } s > s_0}$$

1b) Using the definition, find L.T. for $e^{3t} = f(t)$

$$\mathcal{L}\{e^{3t}\} = \int_0^\infty e^{-st} e^{3t}\, dt \qquad \begin{array}{l} u = (3-s)t \\ du = (3-s)\,dt \end{array}$$

$$= \int_0^\infty e^{-st+3t}\, dt = \frac{1}{3-s}\int_0^\infty e^{(3-s)t}(3-s)\, dt$$

$$= \frac{1}{3-s} e^{(3-s)t}\Big]_0^\infty$$

$$= \frac{1}{3-s} \lim_{B\to\infty} e^{(3-s)t}\Big]_0^B$$

$$= \frac{1}{3-s} \lim_{B\to\infty} e^{(3-s)B} - e^{(3-s)\cdot 0}$$

$\to \infty$ if $s < 3$

$\to 0$ if $s > 3$ ← s_0

$$= \frac{1}{3-s}\left[0 - 1\right] =$$

$$= \frac{1}{s-3} \quad \text{if } s > 3 \leftarrow s_0$$

Now that we have an entry, let's write a table of L. T.'s

$F(s) = \mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt$

$f(t)$	$F(s) = \mathcal{L}\{f(t)\}$	
e^{3t}	$\frac{1}{s-3}$	$s > 3$
e^{at}	$\frac{1}{s-a}$	$s > a$
e^{5t}	$\frac{1}{s-5}$	
e^{-9t}	$\frac{1}{s+9}$	$s > -9$
$\cos(kt)$	$\frac{s}{s^2+k^2}$	$s > 0$
$\sin(kt)$	$\frac{k}{s^2+k^2}$	$s > 0$
$\cos(2\pi t)$	$\frac{s}{s^2+(2\pi)^2}$	
$\sin(6t)$	$\frac{6}{s^2+36}$	
$c_1 f(t) + c_2 g(t)$	$c_1 F(s) + c_2 G(s)$	

linearity property

P7

$y(t)$ $f(t)$	$Y(s)$ $F(s)$
t^n $\rightleftarrows$	$\dfrac{n!}{s^{n+1}}$
$1 = t^0$	$\dfrac{0!}{s^1} = \dfrac{1}{s}$
$8t^3 + 7 \cdot 1 + 5e^{4t}$	$8\dfrac{3!}{s^4} + 7 \cdot \dfrac{1}{s} + \dfrac{5}{s-4}$
$f'(t)$	$sF(s) - f(0)$
$y'(t)$	$sY(s) - y(0)$