Math 212 Lect #17 3-31-10

1st Translation Theorem

$$\mathcal{L}\{e^{at} \cdot f(t)\} = F(s-a) = F(s)\Big|_{s=s-a}$$

$f(t)$	$F(s)$
$e^{at} f(t)$	$F(s-a) = F(s)\Big\vert_{s=s-a}$
$e^{3t} \sin(2t) \rightarrow$	$\dfrac{2}{(s-3)^2+4} = \dfrac{2}{s^2+2^2}\Big\vert_{s=s-3}$
	$= \dfrac{2}{s^2-6s+9+4}$
	$= \dfrac{2}{s^2-6s+13}$
$\leftarrow$	$\dfrac{5s}{s^2+8s+25} = \dfrac{}{\underbrace{s^2+8s+16}-16+25}$

$\frac{8}{2}=4$

$4^2=16$

$$\frac{5s}{(s+4)^2+9}$$

$$\frac{5(s+4-4)}{(s+4)^2+3^2}$$

$$\frac{5(s+4)}{(s+4)^2+3^2} - \frac{20}{(s+4)^2+3^2}$$

$$\frac{5(s}{s^2+3^2} - \frac{20}{s^2+3^2}\Bigg|_{s=s+4}$$

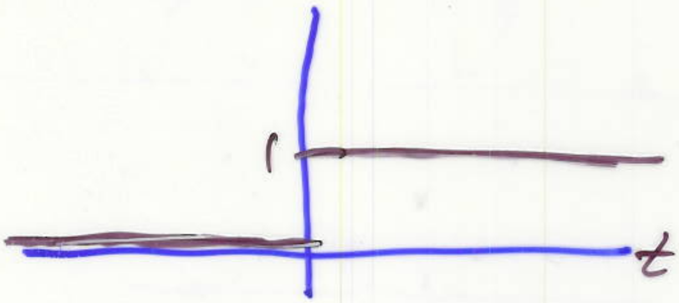
$$\boxed{5\cos(3t) - \frac{20}{3}\sin(3t)} \Leftarrow 5\,\frac{s}{s^2+3^2} - \frac{20}{3}\,\frac{1\cdot 3}{s^2+3^2}\Bigg|_{s=s+4}$$

times e^{-4t}

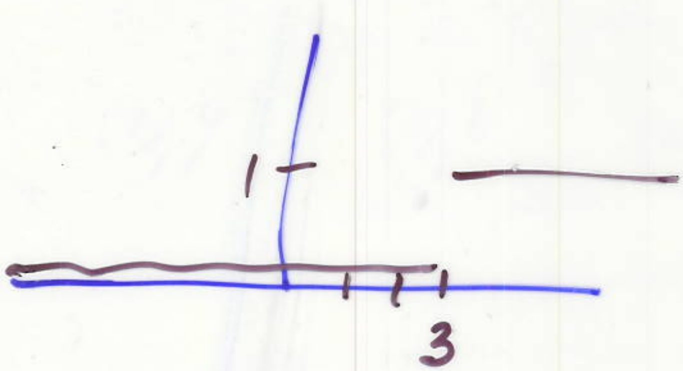
Closing eqn.

$$\mathcal{L}^{-1}\left\{\frac{5s}{s^2+8s+25}\right\} = \mathcal{L}^{-1}\left\{\frac{5s}{(s+4)^2+9}\right\} = \mathcal{L}^{-1}\left\{\frac{5(s+4-4)}{(s+4)^2+9}\right\}$$

2nd Translation Th

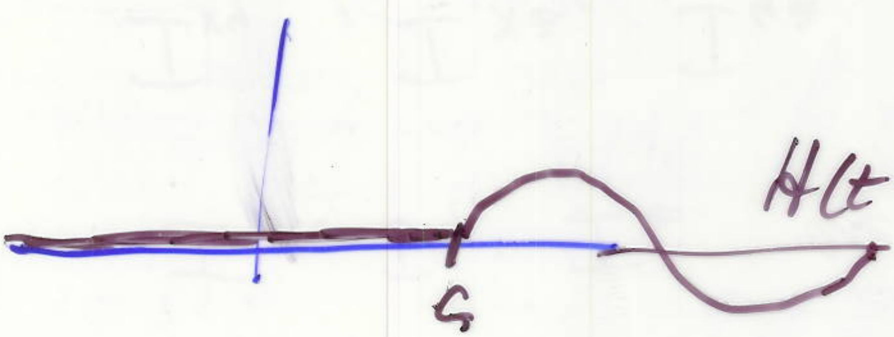


$$H(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases}$$



$$H(t-3) = \begin{cases} 0 & t < 3 \\ 1 & t \geq 3 \end{cases}$$

$H(t)\sin(t)$



$H(t-c)\sin(t-c)$

2^{nd} T Th

$$\mathcal{L}\{H(t-c)\,f(t-c)\} = e^{-cs}\,F(s)$$

$f(t)$	$F(s)$
$H(t-c)\,f(t-c)$	$e^{-cs}\,F(s)$
$H(t-3)\,\sin(t-3)$	$e^{-3s}\cdot\dfrac{1}{s^2+1^2}$

$f(t-c)$

$f(t) = \sin t$

$\sin(A+B) = \sin A\cos B + \cos A\sin B$

$H(t-3)\,\sin t$

$= H([t-3])\,\sin([t-3]+3)$

$= H(t-3)\left[\sin(t-3)\cos 3 + \cos(t-3)\sin 3\right]$

$f(t)\ (\sin t)(\cos 3)$

$$e^{-3s}\left[(\cos 3)\frac{1}{s^2+1} + (\sin 3)\frac{1}{s^2+1}\right]$$