M212 Lect #18 4-5-10

2nd Transl. Th

$$\mathcal{L}\{H(t-c)\, f(t-c)\} = e^{-cs} F(s)$$

$f(t)$	$F(s)$
$H(t-5)\,\dfrac{(t-5)^4}{12}$ ←	$e^{-5s}\,\dfrac{2}{s^5}$
	$= e^{-5s}\,\dfrac{2}{24}\,\dfrac{4!}{s^5}$
	$F(s)$
$f(t) = t^4$ ←	

$t^n \qquad \dfrac{n!}{s^{n+1}}$

To prove most of the 6 theorems we'll use the old trig proof format

$\sec x \cdot \sin x$	$\stackrel{?}{=}$	$\tan x$
$= \dfrac{1}{\cos x} \cdot \sin x$		$=$
$= \dfrac{\sin x}{\cos x}$		
$= \tan x$	$\equiv$	$\tan x$

Proof of 1st Translation Th

$$\mathcal{L}\{e^{at} f(t)\} \overset{?}{=} F(s-a)$$

$$= \int_0^\infty e^{-st} e^{at} f(t)\,dt$$

$$= \int_0^\infty e^{-st+at} f(t)\,dt$$

$$= \int_0^\infty e^{(a-s)t} f(t)\,dt$$

$$= F(s)\Big|_{s=s-a}$$

$$= \mathcal{L}\{f(t)\}\Big|_{s=s-a}$$

$$= \int_0^\infty e^{-st} f(t)\,dt\,\Big|_{s=s-a}$$

$$= \int_0^\infty e^{-(s-a)t} f(t)\,dt$$

$$\doteq \int_0^\infty e^{(a-s)t} f(t)\,dt$$

The Derivative of the Transform Th

$$\mathcal{L}\{t\, f(t)\} = -\frac{d}{ds} F(s)$$

general

$$\mathcal{L}\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n} F(s)$$

$f(t)$	$F(s)$
$t \cdot \sin(3t)$	$-\frac{d}{ds} \frac{3}{s^2+9}$
	$= -\frac{(s^2+9)\,0 - 3(2s)}{(s^2+9)^2}$
	$= \frac{3 \cdot 2s}{(s^2+9)^2}$

$$\frac{5}{2}\, t \cdot \sin(4t) \longleftarrow \frac{20\; 4\cdot 2s}{4\cdot 2\,(s^2+16)^2} = \frac{5}{2} \frac{4\cdot 2s}{(s^2+16)^2}$$

Transform of the Derivative

$$\mathcal{L}\{f'(t)\} = s\,F(s) - f(0)$$

mostly

$$\mathcal{L}\{y'(t)\} = s\,Y(s) - y(0)$$

$$\mathcal{L}\{f''(t)\} = s^2 F(s) - s f(0) - f'(0)$$

$$\mathcal{L}\{f'''(t)\} = s^3 F(s) - s^2 f(0) - s^1 f'(0) - s^0 f''(0)$$

$\vdots$

$$\mathcal{L}\{f^{(n)}(t)\} = s^n F(s) - s^{n-1} f^{(0)}(0) - \cdots - s^0 f^{(n-1)}(0)$$

$$= s^n F(s) - \sum_{k=0}^{n-1} s^{n-1-k} f^{(k)}(0)$$

Ex 1 $y'''' + 3y''' - 5y'' + 2y = e^{6t}$

$y(0) = 5 \quad y'(0) = 6 \quad y''(0) = 7 \quad y'''(0) = 8$

$$1\left(s^4 Y - s^3 5 - s^2 6 - s7 - 8\right)$$
$$+ 3\left(s^3 Y - s^2 5 - s6 - 7\right)$$
$$- 5\left(s^2 Y - s5 - 6\right)$$
$$+ 2\left(Y\right) \qquad = \frac{1}{s-6}$$

Prove 2nd Trans Th

$$\mathcal{L}\{H(t-c)\,\underline{f(t-c)}\} \overset{?}{=} e^{-cs}F(s)$$

$$= \int_0^\infty e^{-st} H(t-c)\, f(t-c)\, dt$$

$$= \cancelto{0}{\int_0^c e^{-st}\, 0\, f(t-c)\,dt} + \int_c^\infty e^{-st}\cdot 1 \cdot f(t-c)\, dt$$

$$= \int_c^\infty e^{-st} f(t-c)\, dt$$

$u = t-c$
$du = dt$
$t = u+c$
$t=c \Rightarrow u=0$
$t\to\infty \Rightarrow u\to\infty$

$$= \int_0^\infty e^{-s(u+c)} f(u)\, du$$

$$= e^{-cs}\int_0^\infty e^{-st} f(t)\, dt$$

$$= \int_0^\infty e^{-cs}e^{-st} f(t)\, dt$$

$$= \int_0^\infty e^{-s(t+c)} f(t)\, dt$$

$t = u$
$dt = dt$

$$\equiv$$