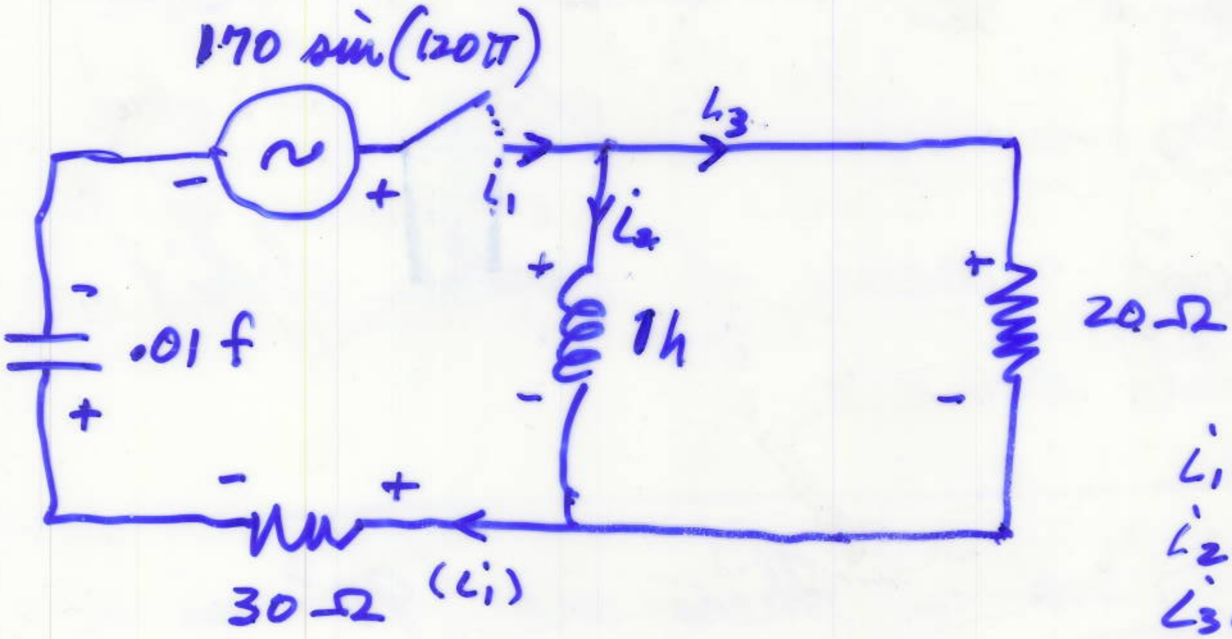
M 212 Lect #21 4-14-10



$i_1(0) = 0$
$i_2(0) = 0$
$i_3(0) = 0$
$q_1(0) = .2$

The sum of the currents entering any point in the circuit equals the sum leaving that point

① $i_1 = i_2 + i_3$ currents

② $1 \cdot i_2' + 30\, i_1 + \frac{1}{.01}\, q_1 = 170 \sin(120\pi t)$ left loop

③ $20\, i_3 - 1 \cdot i_2' = 0$ right loop

omit

$20\, i_3 + 30 i_1 + \frac{1}{.01} q_1 = 170 \sin(120\pi t)$ big loop (redundant)

④ $q_1' = i_1$ charge der

$$1\left(sI_2 - \cancelto{0}{i_2(0)}\right) + 30 I_1 + 100 Q_1 = \frac{170 \cdot 120\pi}{s^2 + (120\pi)^2}$$

$$s Q_1 - \cancelto{.2}{q_1(0)} = I_1$$

$$I_1 = I_2 + I_3$$

$$20 I_3 - 1\left(sI_2 - \cancelto{0}{i_2(0)}\right) = 0$$

Put all 4 eqns in good form

$$\begin{cases} 30 I_1 + s I_2 + 100 Q_1 = \dfrac{170 \cdot 120\pi}{s^2 + (120\pi)^2} \\ -I_1 + s Q_1 = .2 \\ I_1 - I_2 - I_3 = 0 \\ -s I_2 + 20 I_3 = -0 \end{cases}$$

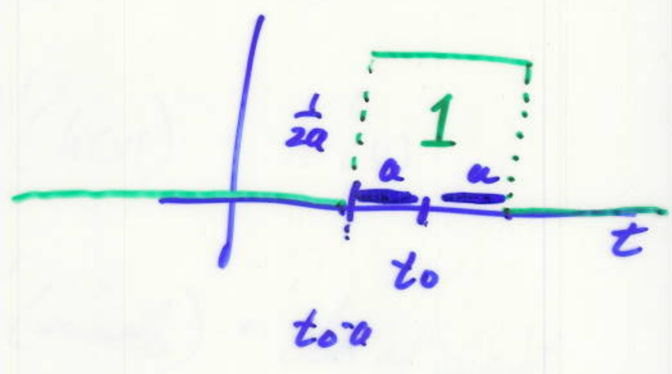
$$I_1 = \frac{\begin{vmatrix} \frac{170 \cdot 120\pi}{s^2 + (120\pi)^2} & s & 0 & 100 \\ .2 & 0 & 0 & s \\ 0 & -1 & -1 & 0 \\ 0 & -s & 20 & 0 \end{vmatrix}}{\begin{vmatrix} 30 & s & 0 & 100 \\ -1 & 0 & 0 & s \\ 1 & -1 & -1 & 0 \\ 0 & -s & 20 & 0 \end{vmatrix}}$$

A few moments on the Dirac Delta Fcn

The regular delta fcn

Unit Table fcn

$$\delta_a(t-t_0) = \begin{cases} 0 \\ \frac{1}{2a} \\ 0 \end{cases}$$



base · ht = 1

$2a \cdot ht = 1$

$ht = \frac{1}{2a}$

Dirac Delta fcn

$$\delta(t-t_0) = \lim_{a \to 0} \delta_a(t-t_0)$$

Unit Impulse Fcn

$$\mathcal{L}\{\delta(t-t_0)\} = e^{-st_0}$$

Chapter 9 Numerical Methods

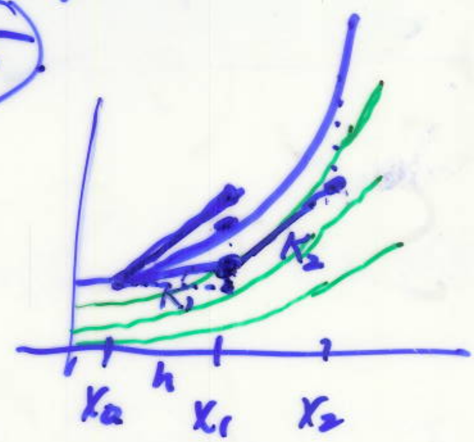
Recall the Euler Method

Given an IVP

$$y' = 1 - x + y = F(x,y) \quad , \quad y(0) = 1$$

Find a numerical soln on the interval [0,1] using $n = 2$ iteration by Euler Method

$$h = \frac{b-a}{n} = \frac{1-0}{2} = .5$$



$$\begin{cases} x_0 = 0 \\ y_0 = 1 \end{cases}$$

$$\begin{cases} x_1 = x_0 + h = 0 + .5 = .5 \\ y_1 = y_0 + h \cdot F(x_0, y_0) = 1 + (.5) \cdot (1 - 0 + 1) = 2 \end{cases}$$

$$\begin{cases} x_2 = x_1 + h = .5 + .5 = 1 \\ y_2 = y_1 + h\,F(x_1, y_1) = 2 + (.5) \cdot (1 - .5 + 2) = 3.25 \end{cases}$$

The num soln is

$$\boxed{\{ (0,1), (.5,2), (1,3.25) \}}$$

Improved Euler Method

$$y' = 1 - x + y \quad , \quad y(0) = 1 \quad \text{on } [0,1]$$

$$n = 2 \qquad h = \frac{1-0}{2} = .5$$

$x_0 = 0$
$y_0 = 1$

$x_1 = x_0 + h = 0 + .5 = .5$

$y_1 = y_0 + h \cdot K$

$= 1 + .5(2.25)$

$= 2.125$

$x_2 = x_1 + h = .5 + .5 = 1$

$y_2 = y_1 + hK$

Numer Soln is

$\{ (0,1) \; (.5, 2.125) \; (1, ?) \}$

$K_1 = F(x_0, y_0) = F(0,1)$

$= 1 - 0 + 1 = 2$

$K_2 = F(x_0 + h, y_0 + h \cdot K_1)$

$= F(0 + .5, 1 + .5(2)) = F(.5, 2)$

$= 1 - .5 + 2 = 2.5$

$K = \frac{K_1 + K_2}{2} = \frac{2 + 2.5}{2}$

$K_1 = F(x_1, y_1) = F(.5, 2.125)$

$\vdots$

$K_2 = F(x_1 + h, y_1 + h K_1)$

$\vdots$

$K = \frac{K_1 + K_2}{2} =$