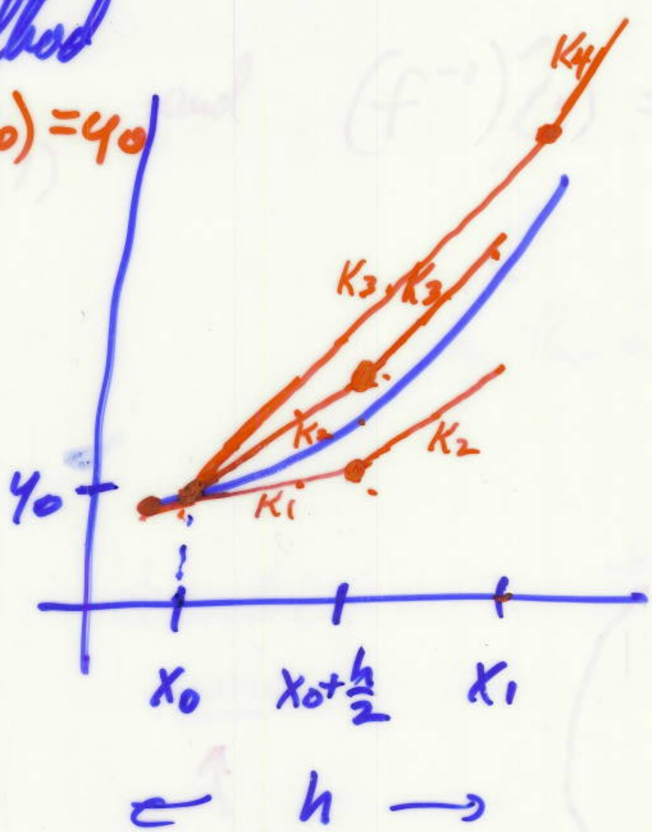
M212 Lect #22 4-19-10

Runge-Kutta Method

$y' = F(x,y)$, $y(x_0) = y_0$



$x_0 = x_0$

$y_0 = y(x_0)$

$x_1 = x_0 + h$

$y_1 = y_0 + hK$

$x_2 = x_1 + h$

$y_2 = y_1 + hK$

$K_1 = F(x_0, y_0)$

$K_2 = F(x_0 + \frac{h}{2}, y_0 + \frac{h}{2}K_1)$

$K_3 = F(x_0 + \frac{h}{2}, y_0 + \frac{h}{2}K_2)$

$K_4 = F(x_0 + h, y_0 + hK_3)$

$$K = \frac{1K_1 + 2K_2 + 2K_3 + K_4}{6}$$

Our two major methods are

① Predictor / Corrector ← Euler, Improved Euler, RK

② Taylor Series $\quad y' = F(x,y)$.

$$f(x) = f(x_0) + f'(x_0)(x-x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2 + \cdots$$

$$= \sum_{k=0}^{\infty} \frac{f^{(k)}(x_0)}{k!}(x-x_0)^k$$

$$y(x) = y(x_0) + y'(x_0)(x-x_0) + \frac{y''(x_0)}{2!}(x_1-x_0)^2 + \cdots$$

$$y(x_1) = y_0 + y'(x_0)(x_1-x_0) + \frac{y''(x_0)}{2}(x_1-x_0)^2$$

$$y_1 \doteq y_0 + F(x_0,y_0)\cdot h + \frac{\frac{d}{dx}F(x,y)}{2}\Bigg|_{\substack{x=x_0\\y=y_0}} h^2$$

$$\begin{cases} x_1 = x_0 + h \\ y_1 = y_0 + h\,F(x_0,y_0) + \dfrac{h^2}{2}\,\dfrac{d}{dx}F(x,y)\Big|_{(x_0,y_0)} \end{cases}$$