M212 Lect #24 4-26-10

We have been solving the IVP numerically

$$y' = F(x,y) = 1-x+y, \quad y(0) = 1,$$

on an interval $[0,1]$

using $n = 2$ iterations

So $h = \frac{1-0}{2} = \frac{1}{2} = .5$

Now use Taylor 2nd order

$$y'(x) = 1-x+y$$

$$y''(x) = 0-1+y'$$

$$= -1+(1-x+y)$$

$$y''(x) = -x+y$$

$$\begin{cases} x_0 = 0 \\ y_0 = 1 \end{cases}$$

$$\begin{cases} x_1 = x_0+h = 0+.5 = .5 \\ y_1 = y_0 + h\,y'(x_0) + \frac{h^2}{2} y''(x_0) \\ \quad = 1 + (.5)(1-0+1) + \frac{(.5)^2}{2}(-0+1) \\ \quad = 1+1+\frac{1}{8} = 2.125 \end{cases}$$

$$x_2 = x_1+h = .5+.5 = 1$$

$$y_2 = y_1 + h\,y'(x_1) + \frac{h^2}{2} y''(x_1)$$

$$= 2.125 + (.5)(1-.5+2.125)$$

$$+ \frac{(.5)^2}{2}(-.5+2.125)$$

$$= 2.125 + 1.3125 + .203125$$

$$= 3.640625$$

Num Soln is

$$\{(0,1), (.5, 2.125), (1, 3.6406)\}$$

Same problem by RK - 4th order

$y' = F(x,y) = 1 - x + y$, $y(0) = 1$, $n = 2$ its

on $[0,1]$, $h = .5$

$x_0 = 0$

$y_0 = 1$

$K_1 = F(x_0, y_0) = 1 - x_0 + y_0$

$= 1 - 0 + 1 = 2$

$x_1 = x_0 + h = 0 + .5 = .5$

$y_1 = y_0 + hK = 1 + (.5)(2.2968)$

$= 2.1484$

$K_2 = F(x_0 + \frac{h}{2}, y_0 + \frac{h}{2} K_1)$

$= F(0 + \frac{.5}{2}, y_0 + \frac{.5}{\cancel{2}} \cancel{2})$

$= F(.25, 1 + .5) = F(.25, 1.5)$

$= 1 - .25 + 1.5 = 2.25$

$K_3 = F(.25, 1 + \frac{.5}{2}(2.25))$

$= F(.25, 1.5625)$

$= 1 - .25 + 1.5625 = 2.3125$

$K = \frac{K_1 + 2K_2 + 2K_3 + K_4}{6}$

$= 2.2968$

$K_4 = F(.5, 1 + hK_3) = F(.5, 2.15625)$

$= F(x_0 + h, y_0 + hK_3)$

$= 1 - .5 + 2.15625$

$= 2.65625$

Can we do numerical methods for systems of IVP's ? Yes!

$$\left\{\begin{array}{ll} x' = F(t, \overbrace{x, y}^{dep}) , & x(t_0) = x_0 \\ y' = G(t, x, y) , & y(t_0) = y_0 \end{array}\right\}$$

$$\begin{cases} t_0 = t_0 \\ x_0 = x_0 = x(t_0) \\ y_0 = y_0 = y(t_0) \end{cases}$$

Use Euler Method on $[a, b]$ using n iterations

$$h = \frac{b-a}{n}$$

$$\begin{cases} t_1 = t_0 + h \\ x_1 = x_0 + h\,F(t_0, x_0, y_0) \\ y_1 = y_0 + h\,G(t_0, x_0, y_0) \end{cases}$$

$$\begin{cases} t_2 = t_1 + h \\ x_2 = x_1 + h\,F(t_1, x_1, y_1) \\ y_2 = y_1 + h\,G(t_1, x_1, y_1) \end{cases}$$

Improved Euler for a system

$$x' = F$$
$$y' = G$$

$$t_0 = t_0$$
$$x_0 = x(t_0)$$
$$y_0 = y(t_0)$$

$$t_1 = t_0 + h$$
$$x_1 = x_0 + hK$$
$$y_1 = y_0 + hL$$

$$K_1 = F(t_0, x_0, y_0)$$
$$L_1 = G(t_0, x_0, y_0)$$

$$K_2 = F(t_0+h,\ x_0+hK_1,\ y_0+hL_1)$$
$$L_2 = G(t_0+h,\ x_0+hK_1,\ y_0+hL_1)$$

$$K = \frac{K_1+K_2}{2} \qquad L = \frac{L_1+L_2}{2}$$

Here's the diagram