M 212 Lect #25 4-28-10

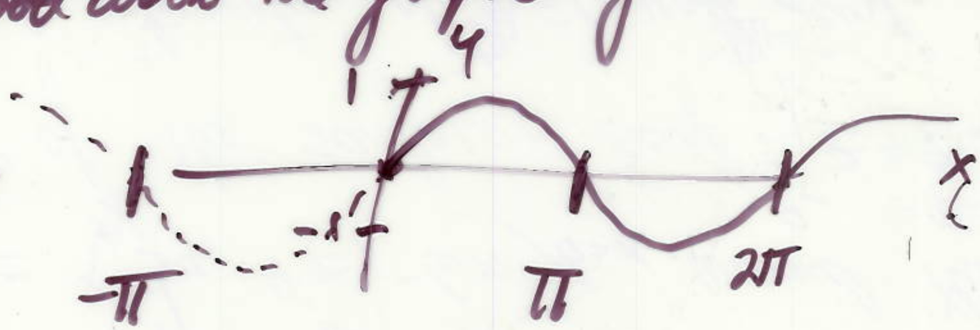
Our last major topics are

<u>Fourier Series</u> and Partial Diff. Eqns
PDE

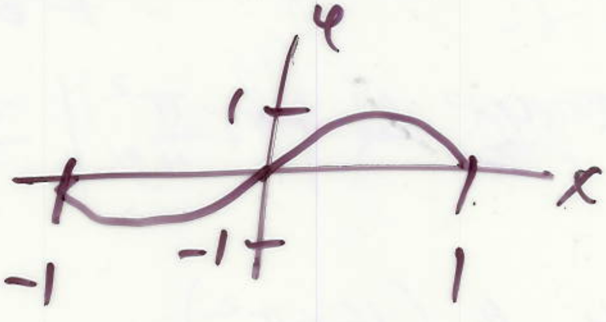
1. Orthog fcns
2. Recall Taylor Ser
3. Do F. S.

Let's get good with the graphs of cos & sin

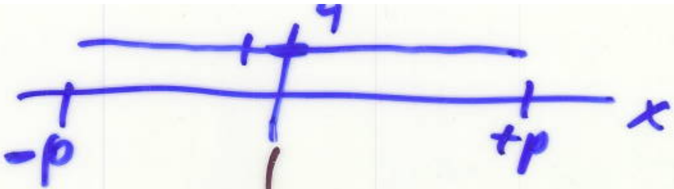
$y = \sin x$



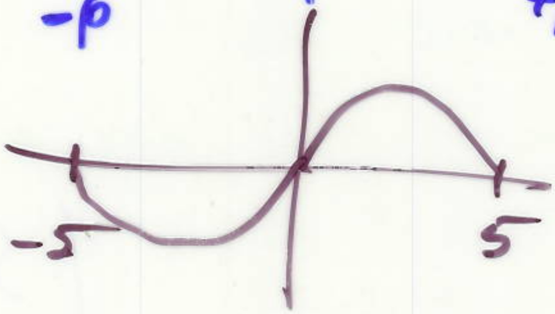
$y = \sin \pi x$



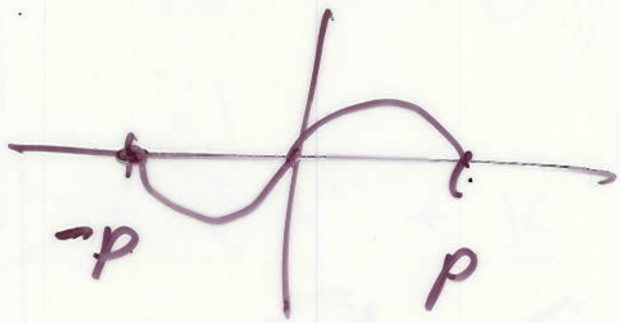
$$y = \cos\frac{0\pi x}{p} = 1$$



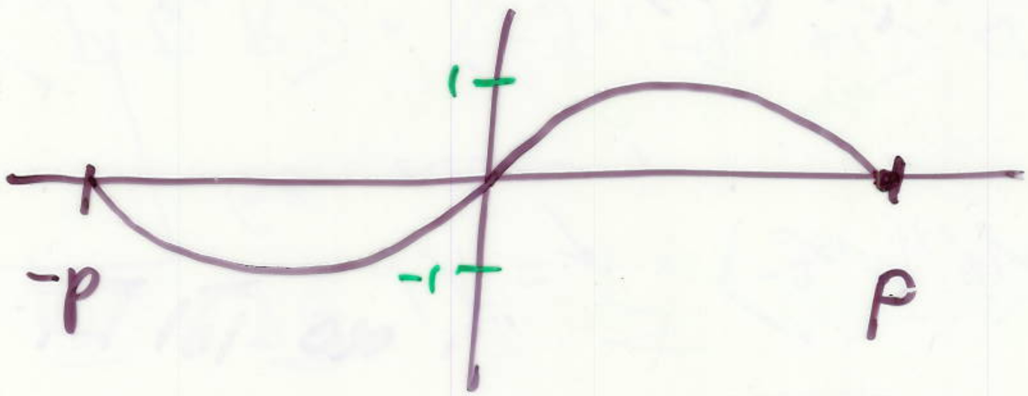
$$y = \sin\frac{\pi x}{5}$$



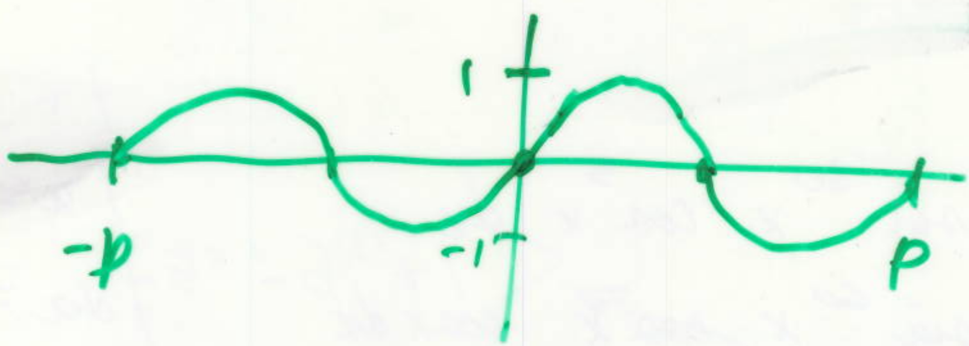
$$y = \sin\frac{\pi x}{p}$$



$$y = \sin\frac{\pi x}{p}$$

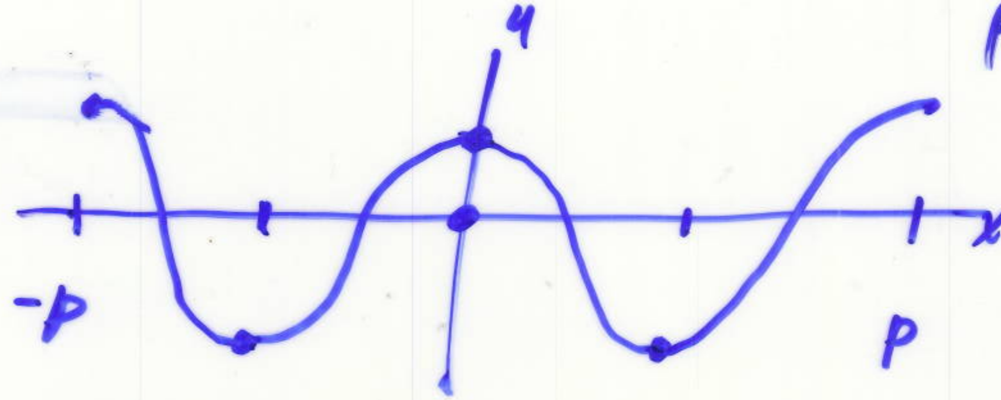


$$y = \sin\frac{2\pi x}{p}$$



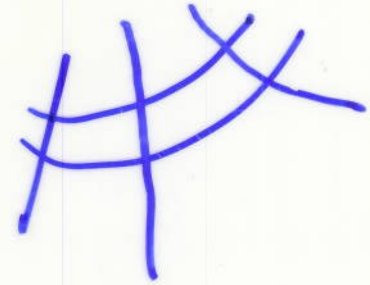
$$y = \sin\frac{5\pi t}{p}$$

$\cos \frac{2\pi x}{p}$



Orthogonal

① DE of OT :



idea was neg recip
to get $\perp$.

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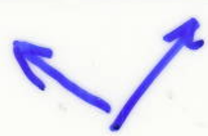
② Two vector



are perp $\perp$ if their dot prod was zero

$$\langle 2, 5 \rangle \cdot \langle 10, -4 \rangle = 20 + (5)(4) = 0$$

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③ New one
Orthog fcns on an interval
w/ a weight fcn.

We say that the fcn. $f(t)$ and $g(t)$ are orthog on an interval $[a,b]$ w/r wt. fcn $w(t)$ if the inner product

$$\langle f,g\rangle = \int_a^b \cancelto{1}{w(t)}\, f(t)\, g(t)\, dt = 0$$

These are the fcn that make up the terms of the Fourier Series $\mathcal{F}(x)$

$$a_0 \cancelto{1}{\cos\frac{0\pi x}{p}} + a_1\cos\frac{1\pi x}{p} + a_2\cos\frac{2\pi x}{p} + a_3\cos\frac{3\pi x}{p} + \cdots + a_n\cos\frac{n\pi x}{p} + \cdots$$

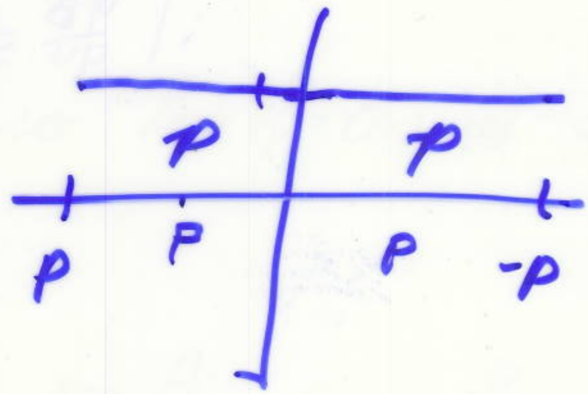
$$\cancel{b_0} \cancelto{0}{\sin\frac{0\pi x}{p}} + b_1\sin\frac{1\pi x}{p} + b_2\sin\frac{2\pi x}{p} + b_3\sin\frac{3\pi x}{p} + \cdots + b_n\sin\frac{n\pi x}{p} + \cdots$$

Algebraically

$$\langle 1, 1\rangle = \int_{-p}^{p} 1\cdot 1\cdot 1\,dx = x\Big]_{-p}^{p} = p-(-p) = 2p$$

Graphically

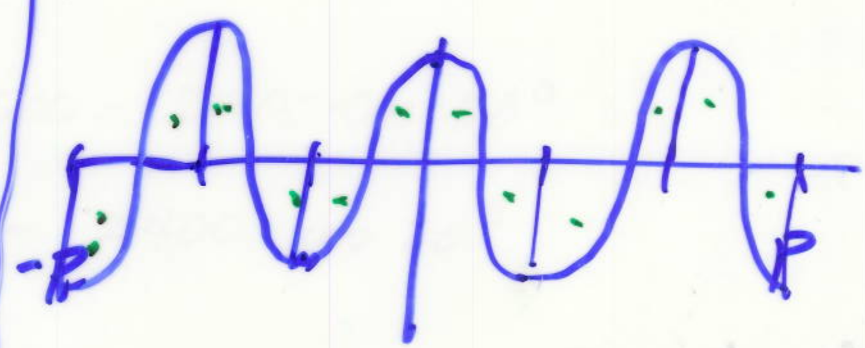
$$\langle 1, 1\rangle = 2p$$



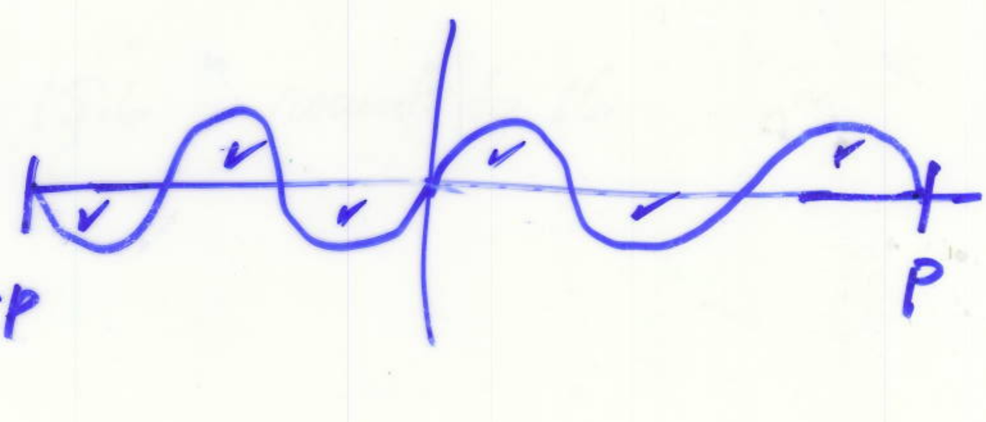
Graphically

$$\left\langle 1, \cos\frac{3\pi x}{p}\right\rangle = 0$$

$$= \int_{-p}^{p} 1\cdot 1\cdot \cos\frac{3\pi x}{p}\,dx$$



$$\left\langle 1, \sin\frac{n\pi x}{p}\right\rangle = 0$$



$$\left\langle \cos\frac{n\pi x}{p}, \sin\frac{m\pi x}{p} \right\rangle = \int_{-p}^{p} \cos\frac{n\pi x}{p} \cdot \sin\frac{m\pi x}{p}\, dx = 0$$

$$\left\langle \cos\frac{n\pi x}{p}, \cos\frac{n\pi x}{p} \right\rangle = p$$

$$\left\langle \sin\frac{m\pi x}{p}, \sin\frac{m\pi x}{p} \right\rangle = p$$

$$\left\langle \cos\frac{m\pi x}{p}, \cos\frac{n\pi x}{p} \right\rangle = 0$$

$$\left\langle \sin\frac{m\pi x}{p}, \sin\frac{n\pi x}{p} \right\rangle = 0$$

So we conclude that
all inner products on our list
are zero except the three types
of fcns with themselves.
So almost all are orthogonal
to each other.

Partial Differential Equations PDE

Indep \ Dep	1	2^+
1	ODE	System of ODEs
2	PDE	System of PDEs

We will solve PDEs to get a "general product solution".
We will apply boundary conditions BCs to the gen prod soln to get a solution to the BVP (Boundary Value Problem)

separation constant

p 10

$$\underbrace{\frac{X'(x)}{X(x)}}_{ODE_1} = k = \underbrace{2\frac{Y'(y)}{Y(y)} - 5}_{ODE_2}$$

p 9

Ex ↓

$u_x - 2u_y + 5u = 0$ where $u = u(x,y)$

$$\frac{\frac{dX}{dx}}{X} = k \qquad \frac{2\frac{dY}{dy}}{Y} - 5 = k$$

a trial soln

We seek a product soln, i.e.

$$\int \frac{dX}{X} = \int k\,dx \qquad 2\frac{dY}{Y} = (k+5)\,dy$$

$$\boxed{u(x,y) = X(x)\,Y(y) = XY}$$

$$\int \frac{dY}{Y(y)} = \int \frac{k+5}{2}\,dy$$

$u_x = X'Y$

$u_y = XY'$

$\ln|X| = kx + c_1$

$\ln|Y| = \frac{k+5}{2}y + c_2$

$|X| = e^{kx}\cdot e^{c_1}$

Plug these into PDE

$|Y| = e^{\frac{k+5}{2}y}\cdot e^{c_2}$

$X = c_3 e^{kx}$

$$\frac{X'Y}{XY} - 2\frac{XY'}{XY} + 5\frac{XY}{XY} = 0 \qquad Y = c_4 e^{\frac{k+5}{2}y}$$

So the general prod soln is

$$u(x,y) = Ce^{kx + \frac{k+5}{2}y}$$

$$\frac{X'}{X} - 2\frac{Y'}{Y} + 5 = 0$$

Let's apply this BC to the gen prod soln

$$u(x,0) = 6e^{3x}$$

$$6e^{3x} = u(x,0) = ce^{kx + \frac{k+5}{2}0}$$

$$6e^{3x} = ce^{kx}$$

So $c = 6$, $k = 3$

$u(x,y) = ce^{kx + \frac{k+5}{2}y}$ becomes

$u(x,y) = 6e^{3x+4y}$, the soln to the BVP