M212 Lect #27 5-5-10

Let find the entire F.S. for the fcn from last time.

$$f(x) = \begin{cases} 1 & -3 \le t < 0 \\ 7 & 0 \le t < 3 \end{cases}$$

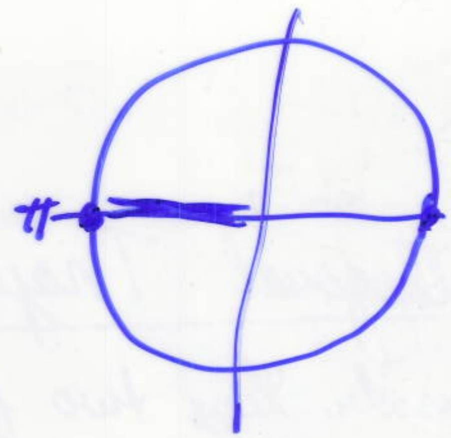
All of the a_n's turn out to be zero. Just believe it.

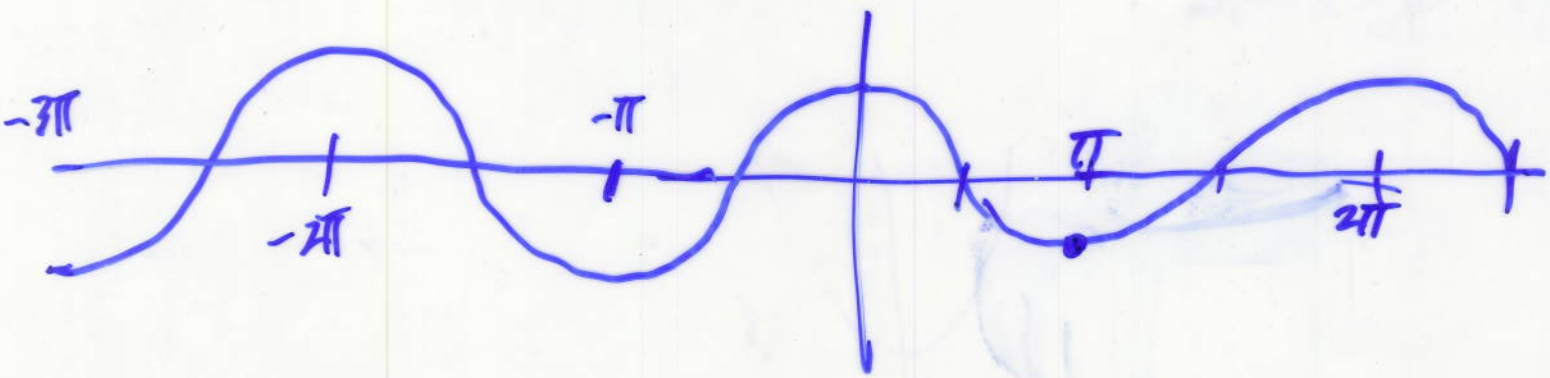
$$b_n = \frac{1}{3}\int_{-3}^{3} \sin\frac{n\pi x}{3}\, f(x)\,dx$$

$$= \frac{1}{\cancel{3}}\frac{\cancel{3}}{n\pi}\left\{\int_{-3}^{0} \sin\left(\frac{n\pi x}{3}\right)\frac{n\pi}{3}\,1\,dx + \int_{0}^{3} \sin\left(\frac{n\pi x}{3}\right)\cdot\frac{n\pi}{3}\,7\,dx\right\}$$

$$= -\frac{1}{n\pi}\left\{1\left(+\cos\frac{n\pi x}{3}\right)\Big]_{-3}^{0} + 7\left(+\cos\frac{n\pi x}{3}\right)\Big]_{0}^{3}\right\}$$

$$= -\frac{1}{n\pi}\left\{1\left(1-\cos(-n\pi)\right) + 7\left(\cos n\pi - 1\right)\right\}$$





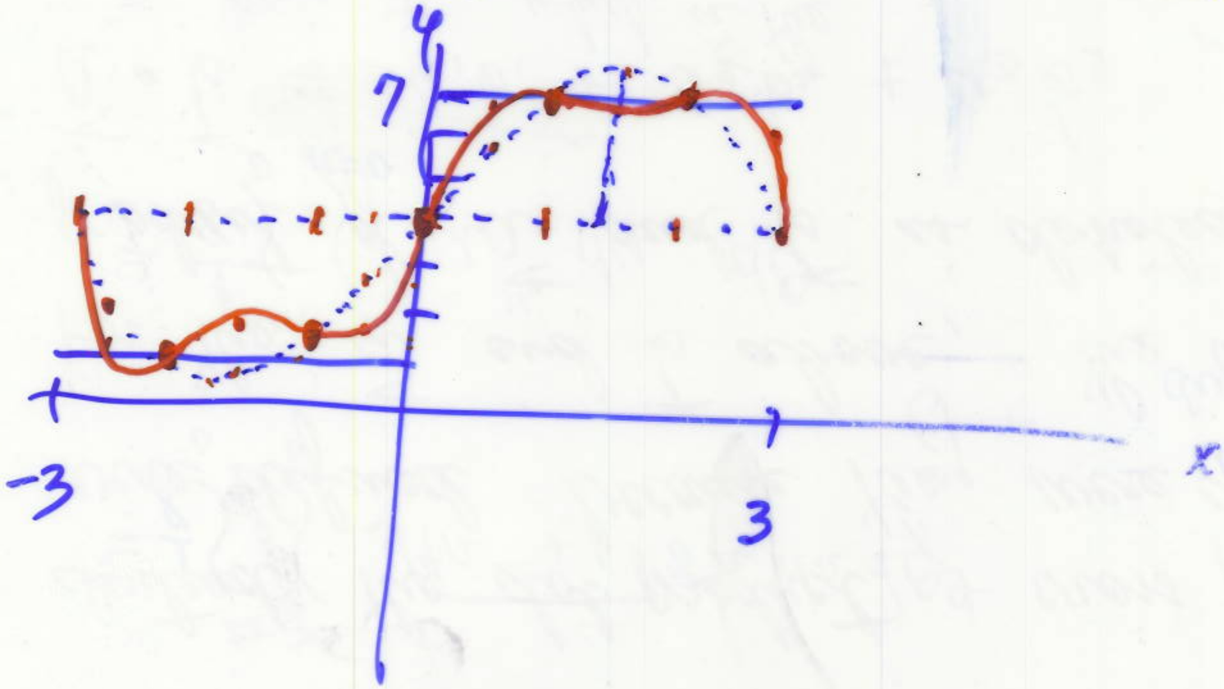
$$b_n = -\frac{1}{n\pi}\left\{1\left(1-(-1)^n\right) + 7\left((-1)^n - 1\right)\right\}$$

$$= -\frac{1}{n\pi}\left\{6\left[(-1)^n - 1\right]\right\} = \frac{6}{n\pi}\left(1-(-1)^n\right)$$

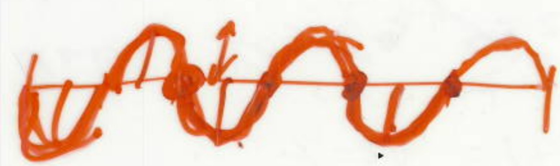
$$b_n = \begin{cases} \frac{2\cdot 6}{n\pi} & \text{for } n \text{ odd} \quad \text{or } \frac{12}{n\pi} \\ 0 & \text{for } n \text{ even} \end{cases}$$

$n=0$ $\quad n=1$ $\quad n=2$ zero $\quad n=3$ $\quad n=4$ zero $\quad n=5$

$$\mathcal{F}_5(x) = 4 + \frac{12}{1\pi}\sin\frac{1\pi x}{3} + \frac{12}{3\pi}\sin\frac{3\pi x}{3} + \frac{12}{5\pi}\sin\frac{5\pi x}{3}$$



$$\mathcal{F}_5(x) \doteq 4 + 3.8\sin\frac{\pi x}{3} + 1.3\sin\frac{3\pi x}{3} + .8\sin\frac{5\pi x}{3}$$



Let's solve this BVP

BC

$$u_x - x u_y + 3u = 0 \quad , \quad u(0,y) = 8e^{5y}$$

$$y \geq 0$$

$$u(x,y) = X(x)\,Y(y)$$

$$u_x = X'Y \qquad u_y = X\,Y'$$

$$\frac{X'Y}{XY} - \frac{x\,XY'}{XY} + \frac{3XY}{XY} = \frac{0}{XY}$$

$$\frac{X'}{X} - x\frac{Y'}{Y} + 3 = 0$$

$$\frac{1}{x}\left(\frac{X'}{X} + 3\right) = \cancel{x}\frac{Y'}{Y}\cdot\frac{1}{\cancel{x}}$$

Vars are Sep'd

$$\frac{1}{x}\left(\frac{X'}{X} + 3\right) = k = \frac{Y'}{Y}$$

$$\cancel{x}\,\frac{1}{\cancel{x}}\left(\frac{dX}{dx\,X}+3\right)=kx \qquad \cancel{dy}\,\frac{dY}{\cancel{dy}\,Y}=k\,dy$$

$$\frac{dX}{dx\,X}+3=kx$$

Variable Sep'd

$$\int\frac{dY}{Y}=\int k\,dy$$

$$\frac{dX}{dx\,X}=kx-3$$

$$\ln|Y|=ky+C_2$$

$e \qquad e$

$$\int\frac{dX}{X}=\int(kx-3)\,dx$$

$$\ln|X|=k\frac{x^2}{2}-3x+C_1$$

$e \qquad e$

$$X=C_3e^{\frac{kx^2}{2}-3x} \qquad Y=C_4e^{ky}$$

$$u(x,y)=Ce^{\frac{kx^2}{2}-3x+ky}$$ is gen prod soln

BC: $u(0,y)=8e^{5y}$

$8e^{5y}=Ce^{ky}$ So $c=8$ $k=5$

Soln to BVP $\boxed{u(x,y)=8e^{\frac{5x^2}{2}-3x+5y}}$

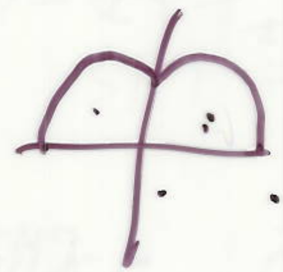
Even & Odd Fcn Operations

even · even = even $x^4 x^6 = x^{10}$

even · odd = odd $x^4 \cdot x^5 = x^9$

odd · odd = even $x^3 \cdot x^5 = x^8$

$$\int_{-p}^{p} \text{even}\, dx = 2\int_{0}^{p} \text{even}\, dx$$



$$\int_{-p}^{p} \text{odd}\, dx = 0$$

$u =$ du

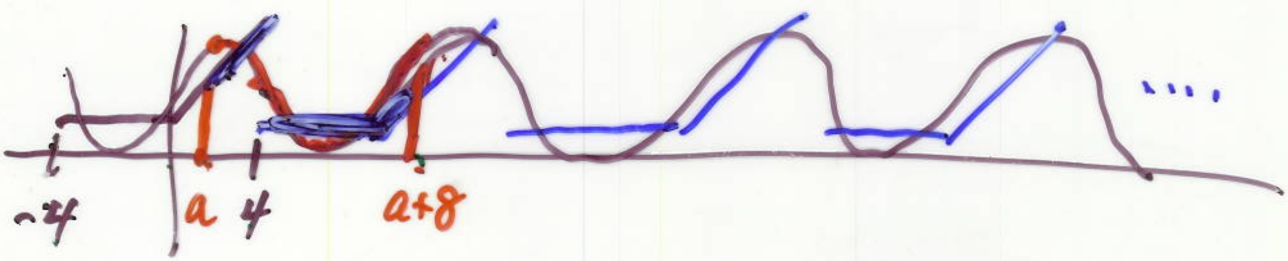
We'll use comb

$$\int_{-p}^{p} \cos\frac{3\pi x}{p} \cdot x^2\, dx = \left(\frac{p}{3\pi}\right)^3 2\int_{0}^{p} \cos\frac{3\pi x}{p} \cdot \left(\frac{3\pi}{p}\right)^2 x^2 \,\frac{3\pi}{p}\, dx$$

$$\int_{-p}^{p} \underbrace{\underbrace{\sin\frac{6\pi x}{p}}_{\text{odd}} \cdot \underbrace{x^4}_{\text{even}}}_{\text{odd}}\, dx = 0$$

Periodic Extension

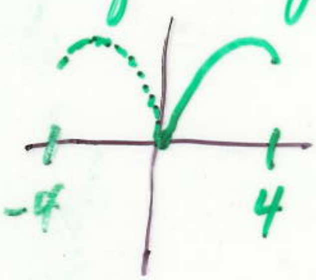
PE



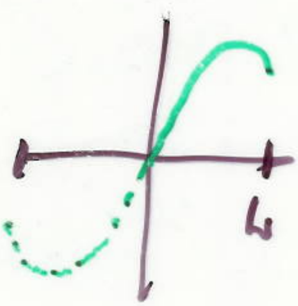
$$f(x+8) = f(x)$$

Half Range Expansion

HRE



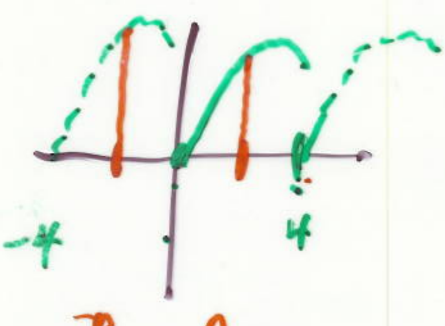
HRE as an even fcn



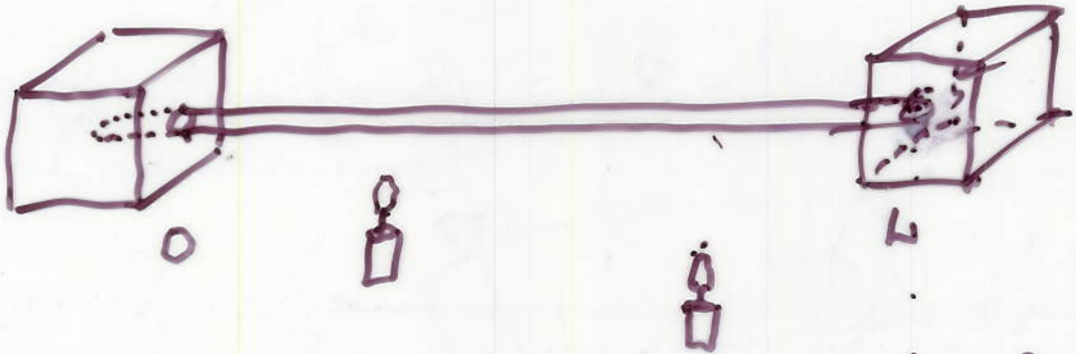
HRE as an odd fcn

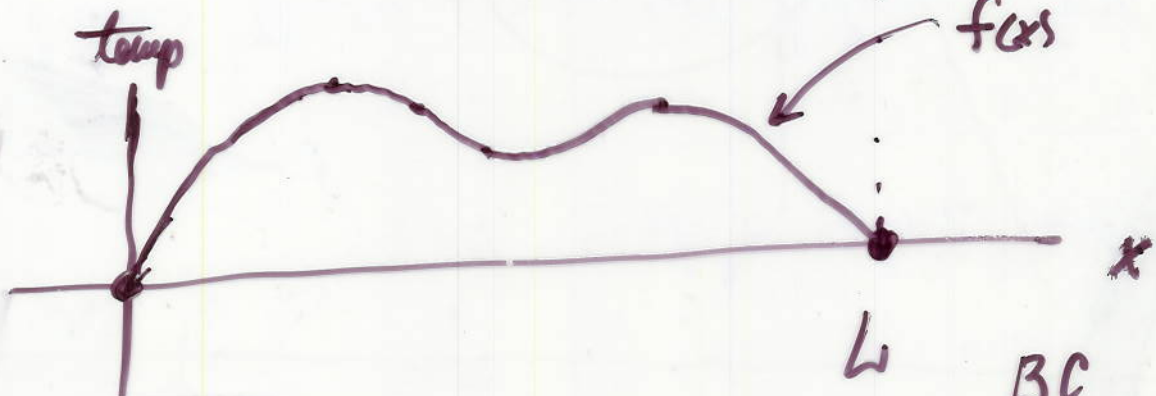
$$a_n = 0$$

$$b_n = \frac{1}{L}\int_{-L}^{L} \sin\frac{n\pi x}{L} f(x)\,dx = \frac{2}{L}\int_{0}^{L} \sin\frac{n\pi x}{L} f(x)\,dx$$



The Heat Eqn





$u(x,t)$ is the temp of the rod at any time $t \geq 0$ and at any place x on the rod $\quad 0 \leq x \leq L$

BC

$$\begin{cases} u(x,0) = f(x) \\ u(0,t) = 0 \\ u(L,t) = 0 \end{cases}$$

$$\boxed{k\,u_{xx} = u_t}$$ is PDE