Math 212 | Problem Set #4 | C.S.Davis

Please begin each numbered problem on a new sheet of paper.

1. a) Define L {f(t)}, the Laplace transform of f(t).

   b) Using the definition above, evaluate L { e^{12t} }

2. Find Laplace transforms for the following:

   a) L { e^{5t} sin(4t) } (use first translation theorem)

   b) L { $\int_0^t e^{6u}\sin(8u)\,du$ } (use convolution theorem)

   c) L { f(t) } where

   $$f(t) = \begin{Bmatrix} e^{5t} & 0 \le t < 3 \\ 7t & 3 \le t \end{Bmatrix}$$ } (use Heaviside step function)

   d) L { t^2 cos(3t) } (use derivative of Lap theorem)

   e) L { g(t) } where

   $$g(t) = \begin{Bmatrix} 9t & 0 \le t < 4 \\ 0 & 4 \le t < 6 \end{Bmatrix}$$ (use periodic function theorem)

   g(t) = g(t+6)

3. Find inverse Laplace transforms for the following:

   a) $L^{-1}\left\{ \dfrac{5s - 8}{s^2 - 8s + 52} \right\}$

   b) $L^{-1}\left\{ \dfrac{4e^{-8s}}{s + 3} \right\}$

   c) $L^{-1}\left\{ \dfrac{6}{s^2 (s-5)} \right\}$, (use convolution theorem)

d) $L^{-1}\left\{\dfrac{e^{-3s}}{(s-6)^4}\right\}$

e) $L^{-1}\left\{\dfrac{50}{(s-3)^2\,(s^2+1)}\right\}$, (Do it by hand, check with Maple and turn both in.)

4. Solve this initial value problem using Laplace transforms.

$y'' - 5y' + 6y = 10\,e^{4t}$, $y(0) = 0$, $y'(0) = 1$

5. Solve this system of linear IVP's using the Laplace transform method. To save time solve for x(t) only.

$$\begin{aligned} x' - 3x + y' &= 0 \qquad & x(0) = 0 \\ 3x' - 9x + 4y' + y &= 0 \qquad & y(0) = 1 \end{aligned}$$

6. Write a system of differential equations with dependent variables i_1, i_2, i_3, and q_1 describing the network below. When the switch is closed all initial currents are zero, and there is an initial charge on the capacitor of 3 coulombs. Take the Laplace transform of each equation and simplify it into "good" form. Finally solve for I_1 using Cramer's rule. Leave the answer in determinant form (4x4 determinants). Do <u>not</u> eliminate any of the four dependent variables.