STATISTICS NOTES FROM VIDEO TAPES

Sessions I through IV

Definition of Statistics

The Science of recording the results of experiments in a concise way so that they yield the maximum amount of information and the making of decisions as a result of this information, especially when the information is incomplete is called statistics

Definition of Probability

The Mathematical Model which makes it possible to make decisions from incomplete data gathered from experiments and yet have some idea of the relative frequency of incorrect decisions is called probability.

Population

The complete collection of elements to be studied. The collection is complete in the sense that it includes all subjects to be studied.

Sample

A sub-collection of elements drawn from the population

Parameter

A numerical measurement describing some characteristic of the population.

Statistic

A numerical measurement describing some characteristic of a sample.

Types of Data

Discrete: The range of values is either a finite set of numbers or a countable infinite set of numbers.

Continuous: The range of values has an uncountable infinite set of numbers.

Random Variable

A random variable (big X) represents a thought or idea about the outcome of an experiment.

A realization of the random variable (little x) is the numerical values that the random variable may assume, or it may be a non-numerical realization such as a color or some other non-quantifiable quality.

Values of Probabilities

Vary from 0 to 1

Sometimes are represented by percentages

Mean--Expected Value--Average

The summation of all realizations of the random variable times their probability of occurrence.

(Represented by the Greek letter μ)

$$\mu = E(X) = \sum_{\forall x} x \cdot P(X = x)$$

Variance

The square of the difference between a realization of a random variable and the expected value of the random variable times the probability of occurrence of the realization of the random variable summed for all values of the random variable.

$$\sigma^2 = \sum_{\forall x} P(X = x) \cdot (x - \mu)^2$$

Fundamental concepts

Experiment -- a process that allows researchers to obtain observations.

Event -- a collection of results of the outcomes of the experiment.

Simple event --an outcome that cannot be broken down any further.

Sample Space -- all possible simple events

Definitions

Experiment: A process that allows researchers to obtain observations.

Event: A collection of results or outcomes of an experiment.

Simple Event: An outcome that cannot be broken down any further.

Sample Space: all possible simple events.

P(A) denotes the probability of event A occurring.

For random variables P(X=x) is the probability that the random variable takes on the value x.

Values of Probability Vary from 0 to 1. Sometimes represented by percentages.

E(X)

Variance(X)

Standard Deviation is the square root of the variance.

Relative Frequency: Perform an experiment a large number of times, count the number of times that an event occurs. P(A) is estimated as the number of times A occurred divided by the number of times an experiment is performed.

Classical Approach: Assume an experiment has n different simple events each of which is equally likely to occur. If an event can occur in s o these n ways, then P(A) = s/n.

Law of Large Numbers: If an experiment is performed over an over for many times, the probability of that event tends to approach its actual Probability.

Random Sample: A sample of n items is a random sample if it is selected in such a way that every possible sample of n items from the population has the same chance of being chosen.

Compliment of an Event: Consists of all outcomes in which A does not occur, denoted by $\bar{A}$. P($\bar{A}$) = 1 - P(A).

Rules for Combining Events

Addition Rule:

P(A or B) = P(A) + P(B) provided that both events cannot occur simultaneously. If this occurs then the events are said to be mutually exclusive.

Not Mutually Exclusive:

P(A or B) = P(A) + P(B) - P(A and B)

Independent Events:

P(A and B) = P(A)xP(B)

Non-Independent (Dependent) Events:

P(A and B) = P(A)xP(B|A)

where P(B|A) represents the probability of B occurring after A has occurred.

Fundamental Theorem of Counting:
If something can be done n different ways and something else can be done m different ways, together we can do both things m times n different ways.

Permutation:
Arrangements of items where order is important.

Combination:
Selection of items where arrangement is not important

1. A computer operator must select 4 jobs from among 10 available jobs waiting to be completed. How many different sequences of running the jobs are possible?

2. An IRS agent must audit 12 returns from a collection of 22 flagged returns. How many combinations are there?

3. A typical combination lock is opened with the correct sequence of 3 numbers between 0 and 49 inclusive. How many different sequences are possible? (A number can be used more than once.)

Are these sequences combinations or are they actually permutations?

4. Each Social Security number is a sequence of 9 digits. How many different Social Security numbers are possible?

5. A Federal Express delivery route must include stops at 7 cities.

a. How many different routes are possible?

b. If the route is randomly selected, what is the probability that the cities will be arranged in alphabetical order?

6. A space shuttle crew has available 10 main dishes, 8 vegetable dishes, 13 desserts, and 3 appetizers. If the first meal includes 2 desserts and 1 item from each of the other categories, how many different combinations for a meal are possible?

For binomial experiments it is necessary to know the values of:

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n, p, x

Then binomial tables can be used to obtain the desired probability.

Multiple Choice Test
100 Questions
4 possible answers per question

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n = 100

p = .25

q = 1 - .25 = .75

X = number of correct answers

Grandmother's Cherry Pies

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n = 20

p = .93

q = .07

If the individual effectiveness in relieving headaches of some advertised pill is 0.8, then how likely would it be that all 10 out of 10 selected users would claim relief?

$$P(X=10)=\binom{10}{10}(.8)^{10}(.2)^{0}$$

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p = .107

Binomial Table

For Binomial Random Variables ONLY

$\mu = E(X) = np$

For Binomial Random Variables ONLY

$\sigma^2 = E(X-\mu)^2 = npq$

$\sigma = \sqrt{npq}$

For Binomial Random Variables ONLY

$\sigma = \sqrt{npq}$

Summary

Binomial Random Variable

$P(X=x) = \binom{n}{x}(p)^x (1-p)^{n-x}$

$\mu = E(X) = np$

$\sigma^2 = E(X-\mu)^2 = npq$

$\sigma = \sqrt{npq}$

Binomial Experiment:

1. A fixed number of trials.
2. The trials are independent.
3. Each trial produces only success or failure.
4. The probability of success from one trial to another must remain the same.

Consider A Binomial Experiment:

Let $n = 8$

Let $x = 3$

Let $p = .7$

Then $q = .3$

How do we calculate

$P(X = 3)$?

With 3 successes this would imply that there are 5 failures. The probability of this occurring is:

$$(.7)(.7)(.7)(.3)(.3)(.3)(.3)(.3)$$

But there is one other requirement! These 3 successes could be obtained on any 3 of the 8 trials, so we have to count the ways that we can get 3 successes out of 8 trials. This is just the number of combinations of 3 things from among 8.

$$P(X = 3) = \binom{8}{3}(.7)^3(.3)^5$$