

Graded Homework Set 4 -- Calculus I -- Math191 (Page references in red)

1. A particle moves along the curve  $y = \sqrt{1+x^3}$ . As it reaches the point (2, 3), the y-coordinate is increasing at the rate of 4 cm/sec. How fast is the x-coordinate of the point changing at that instant? (See examples in section 3.8)

2. An airplane is  $x = 40$  miles away from an airport at a height of  $h$  miles. A radar at the airport detects that the distance  $s(t)$  between the plane and airport is changing at the rate of  $s'(t) = -240$  miles per hour. What is the speed  $x'(t)$  of the airplane if the height is  $h = 4$ ? (Draw a picture and use the Pythagorean Theorem to solve the Problem.) (See examples in section 3.8)

3. Find the critical numbers and determine whether they represent a local maximum, local minimum, or neither if: (See section 4.1)

a.  $y = f(x) = x^3 - 3x^2 + 6x$

b.  $y = f(x) = \sin(x) \cos(x)$  on the interval  $[0, 2\pi]$ .

4. A section of a roller coaster is in the shape of  $y = x^5 - 4x^3 - x + 10$ , where  $x$  is between -2 and 2. Find all relative extrema and explain what portions of the roller coaster they represent. (See section 4.1)

5. a. Show that the function  $y = f(x) = \sqrt[3]{x}$  satisfies the hypotheses of the Mean Value Theorem on the interval  $[0, 1]$ .

b. Find all numbers  $c$  that satisfy the conclusion of the Mean Value Theorem. (See section 4.2),

6. If  $y = f(x) = \frac{x-1}{x^2-x-2}$

a. For what values of  $x$  is  $f(x)$  increasing?

b. For what values of  $x$  is  $f(x)$  decreasing?

c. What are the coordinates of any relative maximum or minimum values for the function?

d. For what values of  $x$  is the function concave down?

e. Find  $\lim_{x \rightarrow \infty} f(x)$ .

f. Graph the function using the above information.

(See section 4.5)

7. a. Show that  $y = f(x) = \frac{x}{\sqrt{25 + x^2}}$  is an increasing function. (See section 4.7)

b. If  $s(t) = -t^3 + 9t^2 - 23t + 15$  represents the position of a particle and  $ds/dt$  represents its velocity, on the interval  $t = 0$  to  $t = 4$ , at what time does the particle reach its maximum velocity?

(See section 4.7)

8. A company needs to run an oil pipeline from an oil rig 25 miles out to sea to a storage tank that is 5 miles inland. The shoreline runs East/West and the tank is 8 miles east of the rig. Assume that it costs \$50 thousand per mile to construct the pipeline underwater and \$20 thousand per mile to construct the pipeline on land. The pipeline will be built in a straight line from the rig to a selected point on the shoreline, then in a straight line to the storage tank. What point on the shoreline should be selected to minimize the total cost of the pipeline?

(See example 4 page 260)

9. Use Newton's method to find  $x_3$ , the third approximation to the root of

$\frac{1}{3}x^3 + \frac{1}{2}x^2 + 3 = 0$  with the value of  $x_1 = -3$ .

(See example 1 page 270)