

Free Fall Problem Set

In each of these problems, ignore the possibility of air resistance. Assume that acceleration due to gravity is downward and equal to g .

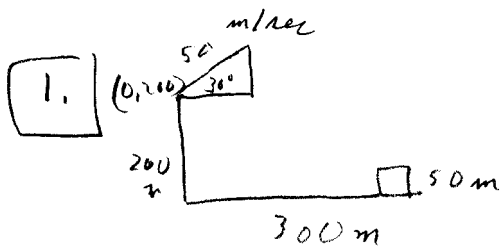
1. A cannon sitting atop of a 200m cliff shoots a projectile at a velocity of 50 m/s and at an angle of 30° above the horizontal. A building 50m tall sits 300m from the base of the cliff. Does the projectile strike the building? (Ignore the width of the building).

2. A lacrosse player 80m. from an open goal throws a ball at an angle of 25° above the horizontal with a velocity of 20 m/s. Does the ball enter the goal in the air? Assume that the ball leaves the stick 3 m above the ground and that a lacrosse goal is 2 m high.

3. An F-18 is flying at 200 m/s at an altitude of 1500m and at an angle of 5° below the horizontal when it drops a bomb. There is a 300 m building 3000 m from the point below the F-18 when it drops its bomb. Does the bomb hit the building? (Ignore the width of the building).

4. In Fenway Park, the Green Monster is a wall approximately 11.3 m tall and 94 m from home plate along third base line. A ball was hit at an angle of 30° along the third base line and barely cleared the Green Monster. At what velocity did the ball leave the bat? Assume that batter hit the ball 1 m above the ground.

FREE FALL SOLUTIONS



$$\vec{v}(0) = \langle 50 \cos 30, 50 \sin 30 \rangle = \langle 43.3, 25 \rangle$$

$$\vec{r}(0) = \langle 0, 200 \rangle$$

$$\vec{a}(t) = \langle 0, -9.8 \rangle$$

$$\vec{v}(t) = \int \vec{a}(t) dt = \langle c_1, -9.8t + c_2 \rangle$$

$$\text{since } \vec{v}(0) = \langle 43.3, 25 \rangle = \langle c_1, 0 + c_2 \rangle$$

$$\vec{v}(t) = \langle 43.3, -9.8t + 25 \rangle$$

$$\vec{r}(t) = \int \vec{v}(t) dt = \langle 43.3t + k_1, -\frac{9.8t^2}{2} + 25t + k_2 \rangle$$

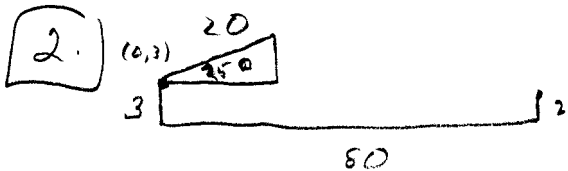
$$\text{since } \vec{r}(0) = \langle 0, 200 \rangle = \langle k_1, k_2 \rangle$$

$$\text{then } \vec{r}(t) = \langle 43.3t, -4.9t^2 + 25t + 200 \rangle$$

$43.3t$ is x component of distance, so $43.3t = 300 \rightarrow t = 6.9$

$-4.9t^2 + 25t + 200$ is y component of height so at $t = 6.9$

$-4.9(6.9)^2 + 25(6.9) + 200 = 139.2\text{m}$ too high, building is missed.



$$\vec{v}(0) = \langle 20 \cos 25, 20 \sin 25 \rangle = \langle 18.13, 8.45 \rangle$$

$$\vec{r}(0) = \langle 0, 3 \rangle$$

$$\vec{a}(t) = \langle 0, -9.8 \rangle \quad \vec{v}(t) = \int \vec{a}(t) dt = \langle c_1, -9.8t + c_2 \rangle \text{ so at } t=0$$

$$\langle c_1, c_2 \rangle = \langle 18.13, 8.45 \rangle$$

$$\vec{v}(t) = \langle 18.13, -9.8t + 8.43 \rangle$$

$$\vec{r}(t) = \int \vec{v}(t) dt = \langle 18.13t + k_1, -4.9t^2 + 8.43t + k_2 \rangle \text{ and at } t=0$$

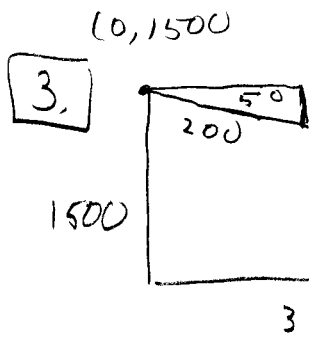
$$\langle k_1, k_2 \rangle = \langle 0, 3 \rangle$$

$$\vec{r}(t) = \langle 18.13t, -4.9t^2 + 8.43t + 3 \rangle$$

$18.13t$ is x component of position and $18.13t = 80 \quad t = \frac{80}{18.13} = 4.41$

the height of the ball is $-4.9t^2 + 8.43t + 3$ with $t = 4.41$

so height at $80\text{m} - 55\text{m}$. Didn't go in, in fact hit the ground when $-4.9t^2 + 8.43t + 3 = 0$ and t is the 2.02 with distance of 36m .



$$v(0) = \langle 200 \cos 5^\circ, -200 \sin 5^\circ \rangle$$

$$= \langle 199.2, -17.43 \rangle$$

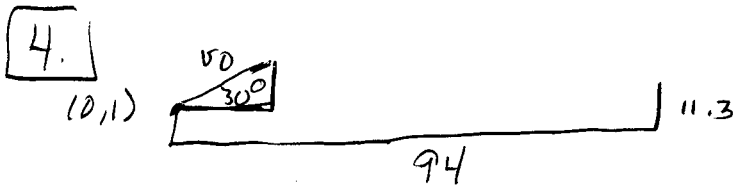
$$r(0) = \langle 0, 1500 \rangle$$

$$\vec{a}(t) = \langle 0, -9.8 \rangle \quad v(t) = \langle 199.2, -9.8t - 17.43 \rangle$$

$$r(t) = \langle 199.2t, -4.9t^2 - 17.43t + 1500 \rangle$$

$$199.2t = 3000 \rightarrow t = 15 \text{ and}$$

with $h(t) = -4.9t^2 - 17.43t + 1500$ $h(15) = 136 \text{ m}$
 so building is hit.



$$v(0) = \left\langle \frac{v_0 \sqrt{3}}{2}, \frac{v_0}{2} \right\rangle$$

$$r(0) = \langle 0, 1 \rangle$$

$$a(t) = \langle 0, -9.8 \rangle \quad v(t) = \left\langle \frac{v_0 \sqrt{3}}{2}, -9.8t + \frac{v_0}{2} \right\rangle$$

$$r(t) = \left\langle \frac{v_0 \sqrt{3}}{2} t, -4.9t^2 + \frac{v_0}{2} t + 1 \right\rangle$$

\nearrow x component of dist

$$\frac{v_0 \sqrt{3}}{2} t = 94 \rightarrow t = \frac{108.55}{v_0} \text{ then } -4.9t^2 + \frac{v_0}{2} t + 1 \text{ at } t = \frac{108.55}{v_0}$$

gives $-4.9 \left(\frac{108.55}{v_0} \right)^2 + \frac{v_0}{2} \left(\frac{108.55}{v_0} \right) + 1 = 11.3$

$$\text{or } -\frac{57737}{v_0^2} + 54.28 + 1 = 11.3 \quad \text{or } \frac{57737}{v_0^2} = 43.98$$

$$\text{or } v_0^2 = \frac{57737}{43.98} \quad \text{and } v_0 = \sqrt{\frac{57737}{43.98}} = 36.23 \text{ m/sec}$$