Name SOLUTIONS

SM223 Performance Opportunity # 1 10 Sep 2008
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1. Find the values of x such that the vectors < 3, 2, x > and < 2x, 4, x > are perpendicular.

$6x + 8 + x^2 = 0$ $\quad x^2 + 6x + 8 = 0$

$(x+4)(x+2) = 0 \rightarrow x = -2$ or -4

2. Find the angle between the vectors < 1, 0 > and < 1, 1 >. (Show your work)

$\frac{\langle 1,0\rangle \cdot \langle 1,1\rangle}{\sqrt{1} \cdot \sqrt{2}} = \cos\theta \qquad \cos\theta = \frac{1}{\sqrt{2}} \rightarrow \theta = \pi/4$ or 45°

3. Find the equation of the line through (2, - 2, 4) and perpendicular to the plane – x + 2y + 5z = 12.

$\frac{x-2}{-1} = \frac{y+2}{2} = \frac{z-4}{5}$

Consider the points A = (1, 1, -2) B = (-3, - 4, 2) and C = (-3, 4, 1). Find:

4. The vector AB. $= \langle -3-1, -4-1, 2-(-2)\rangle = \langle -4, -5, 4\rangle$

5. The vector AC $= \langle -3-1, 4-1, 1-(-2)\rangle = \langle -4, 3, 3\rangle$

6. The equation of the plane containing these three points.

$\times \begin{matrix} -4 & -5 & 4 \\ -4 & 3 & 3 \end{matrix} \rightarrow \langle -27, -4, -32\rangle$

$-27(x-1) - 4(y-1) - 32(z+2) = 0$

$-27x - 4y - 32z = 33$

Consider the vectors **A** = < 1, 0, 4 > and **B** = < 1, - 4, 2 >

7. Find a unit vector perpendicular to the two vectors

$\times \begin{matrix} 1 & 0 & 4 \\ 1 & -4 & 2 \end{matrix} \rightarrow \langle 16, 2, -4\rangle \qquad \frac{\langle 16, 2, -4\rangle}{\sqrt{256+4+16}}$

8. Find the angle between the two vectors.

$\frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|} = \frac{9}{\sqrt{17}\sqrt{21}} = \cos\theta \quad \therefore \theta = 61.6^\circ$ or 1.07 rad

9. A non-zero vector perpendicular to vector **A**.

MANY POSSIBILITIES

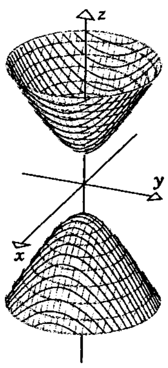
$\langle -4, 0, 1\rangle$ or $\langle 4, 0, -1\rangle$ etc.

10. Find comp$_B$**A**.

$= \frac{\vec{A}\cdot\vec{B}}{|\vec{B}|} = \frac{9}{\sqrt{21}} = 1.96$

Consider the 6 quadric surfaces shown below:

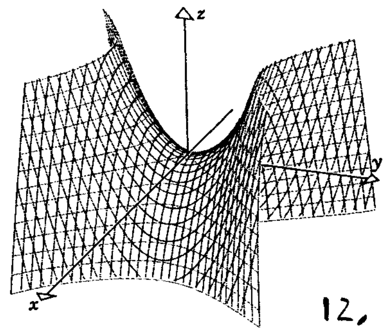
HYPERBOLOID OF TWO SHEETS



11.

E

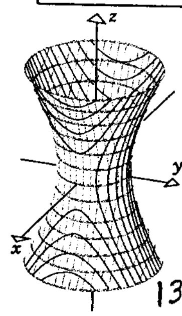
HYPERBOLIC PARABOLOID



12,

F

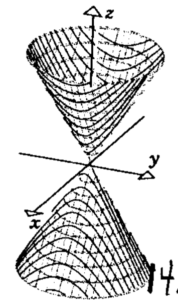
HYPERBOLOID OF ONE SHEET



13,

C

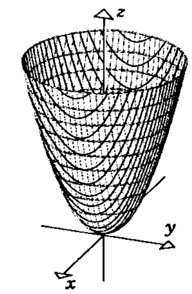
ELLIPTIC CONE



14.

B

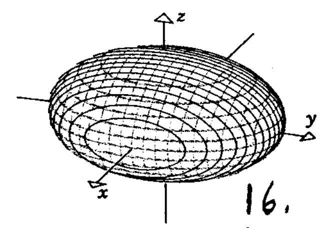
ELLIPTIC PARABOLOID



15,

D

ELLIPSOID



16.

A

Match these graphs with the equations shown below:

A. $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ 16

B. $z^2=\frac{x^2}{a^2}+\frac{y^2}{b^2}$ 14

C. $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$ 13

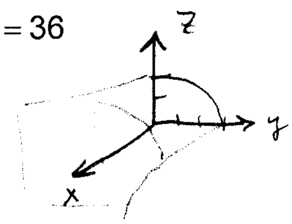
D. $z=\frac{x^2}{a^2}+\frac{y^2}{b^2}$ 15

E. $-\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ 11

F. $z=\frac{y^2}{b^2}-\frac{x^2}{a^2}$ 12

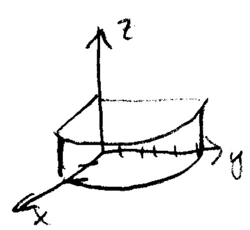
Sketch the first octant view of the 3 dimensional graphs whose equations are:

17. $9z^2+4y^2-9x^2=36$



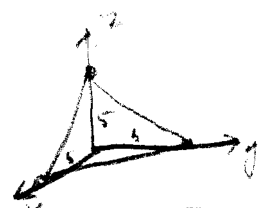
HYPERBOLOID (ONE SHEET) OPEN ON X AXIS

18. $\frac{x^2}{4}+\frac{y^2}{25}=1$



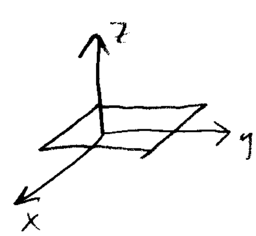
CYLINDER OPEN ON Z AXIS

19. x + y + z = 5



PLANE

20. z = 1



PLANE