Name SOLUTIONS

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SM223 Performance Opportunity # 2a 26 Sep 2008

1. If $\mathbf{r}(t) = \langle t^2 + 2, 4, t^3 \rangle$, find

   a. $\mathbf{v}(t) = \langle 2t, 0, 3t^2 \rangle$

   b. $\mathbf{a}(t) = \langle 2, 0, 6t \rangle$

2. If $\mathbf{a}(t) = \langle t, 0, -16 \rangle$ and $\mathbf{v}(0) = \langle 12, -4, 0 \rangle$ and $\mathbf{r}(0) = \langle 5, 0, 2 \rangle$, find:

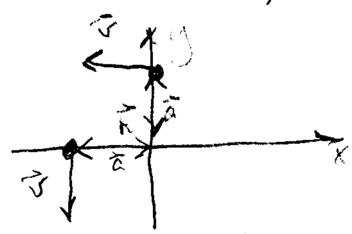
   a. $\mathbf{v}(t) = \langle \frac{t^2}{2} + 12, -4, -16t \rangle$ $\quad \langle c_1, c_2, c_3 \rangle = \langle 12, -4, 0 \rangle$

   b. $\mathbf{r}(t) = \langle \frac{t^3}{6} + 12t + 5, -4t, -8t^2 + 2 \rangle$

3. If $\mathbf{r}(t) = \langle \cos(t), \sin(t) \rangle$, (Note $x^2 + y^2 = 1$ is a circle of radius 1) find

   a. $\mathbf{v}(t) = \langle -\sin t, \cos t \rangle$

   b. $\mathbf{a}(t) = \langle -\cos t, -\sin t \rangle$



   c. plot the values of $\mathbf{r}(t)$, $\mathbf{v}(t)$, and $\mathbf{a}(t)$ for $t = \pi/2$ and π.

4. If $\mathbf{r}(t) = \langle t, 4t - 1, 2 - 6t \rangle$, find the arc length along this curve from $t = 0$ to $t = 2$.

$$\int_0^2 \sqrt{1 + 16 + 36}\, dt = \int_0^2 \sqrt{53}\, dt = 2\sqrt{53} = 14.56$$

5. A tennis serve is struck horizontally from a height of 8 feet, i.e. with $\bar{r}(0) = < 0 , 8 >$, with initial speed of 120 feet per second. For the serve to count, i.e. be "in," it must clear a net which is 39 feet away and 3 feet in height, and must land before the service line 60 feet away.

a. Find a vector function for the position of the ball using calculus.

$\bar{a} = \langle 0, -32 \rangle$

$\bar{v} = \langle 120, -32t \rangle$

$\bar{r} = \langle 120t, -16t^2 + 8 \rangle$

b. How high is the ball when it passes over the net, or hits the net?

$120t = 39$

$t = .325$

$-16(.325)^2 + 8 = 6.31'$

c. How far from the server does the ball land? Is the serve "in" or "out?"

$-16t^2 + 8 = 0$

$t^2 = \frac{1}{2}$

$t = .707$

$120(.707) = 84''$

Out