Name____

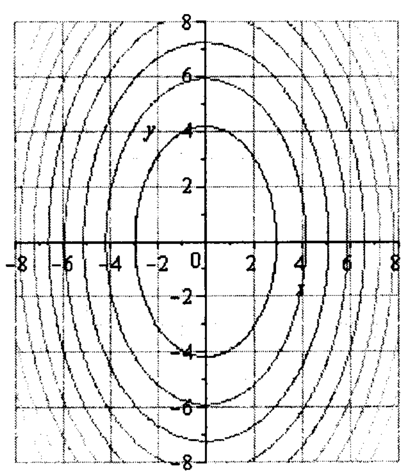
SM223 (Ehler)   Performance Opportunity 3   29 October 2008

1. If $f(x, y, z) = x^2y - z$, defines a given surface, consider the point (2, 1, 4).

   a. Find the equation of the tangent plane to the surface at the point.

   b. Find the equation of the normal line to the surface at the point.

2. Consider the function whose contour plot is shown below. $z = f(x, y) = \frac{x^2}{2} + \frac{y^2}{4}$

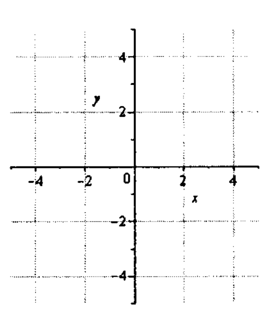


   a. Find the gradient of z.

   b. Evaluate the gradient at the point (-2, 3).

   c. Sketch this gradient vector on the graph above at the point (-2, 3).

3. Sketch a contour plot for $z = \frac{y}{x}$ with z = 1, 2 and 3



4. On a certain mountain, the elevation z above a point (x, y) in a horizontal xy-plane that lies at sea level is $z = 2000 - 2x^2 - 4y^2$ feet. The positive x-axis points East, and the positive y-axis points North. A climber is at the point (-20, 5, 1100).

a. If the climber uses a compass reading to walk due West, will he or she begin to ascend or descend?

b. If the climber uses a compass reading to walk Northeast, will he ascend or descend and at what rate?

c. In what direction should the climber walk to travel a level path?

5. If $z = 4x^2 - 2y + 7x^4y^5$, find:

a. $\frac{\partial z}{\partial x}$

b. $\frac{\partial^2 z}{\partial x^2}$

c. $\frac{\partial^2 z}{\partial y \partial x}$

6. If $z = xe^y - ye^x$, find the directional derivative at the point (0, 0) in the direction $< 4, -3 >$.

7. a. Find the derivative(s) of the function $z = 3x^2y^3$ if $x = t^4$ and $y = t^2$.

b. Find the derivative(s) of the function $z = \cos(x)\sin(y)$ if $x = u - v$ and $y = u^2 + v^2$.

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1 $f(x,y,z) = x^2y - z$

$(2,1,4)$ $\nabla f = \langle 2xy, x^2, -1 \rangle$

(a) TANGENT PLANE

$\nabla f = \langle 4, 4, -1 \rangle$

$4(x-2) + 4(y-1) - 1(z-4) = 0$

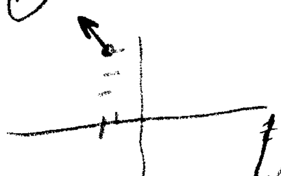
(b) NORMAL LINE

$\frac{x-2}{4} = \frac{y-1}{4} = \frac{z-4}{-1}$

2. $z = \frac{x^2}{2} + \frac{y^2}{4}$

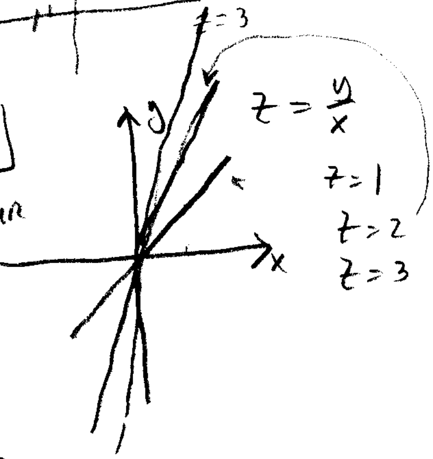
(a) $\nabla z = \langle x, \frac{y}{2} \rangle$

(b) $\langle -2, \frac{3}{2} \rangle$



3.

CONTOUR PLOT



4. $z = 2000 - 2x^2 - 4y^2$

$(-20, 5, 1100)$

$\nabla z = \langle -4x, -8y \rangle$

$= \langle 80, -40 \rangle$

(a) $\langle 80, -40 \rangle \cdot \langle -1, 0 \rangle$

$= -80$ DESCEND

(b) $\langle 80, -40 \rangle \cdot \langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$

$= \frac{40}{\sqrt{2}}$ ASCEND AT RATE

4c. $\langle 80, -40 \rangle \cdot \langle a, b \rangle = 0$

$\therefore \langle 1, 2 \rangle$ or $\langle -1, -2 \rangle$

5. $z = 4x^2 - 2y + 7x^4y^5$

(a) $\frac{\partial z}{\partial x} = f_x = 8x + 28x^3y^5$

(b) $\frac{\partial^2 z}{\partial x^2} = f_{xx} = 8 + 84x^2y^5$

(c) $\frac{\partial^2 z}{\partial y \partial x} = f_{xy} = 140x^3y^4$

6. $z = xe^y - ye^x$ $(0,0)$ $\langle 4, -3 \rangle$

$\nabla z = \langle e^y - ye^x, xe^y - e^x \rangle$

at $(0,0) \to \langle 1, -1 \rangle$

DIR DER $\to \langle 1, -1 \rangle \cdot \langle \frac{4}{5}, -\frac{3}{5} \rangle$

$= 7/5$

7. (a) $z = 3x^2y^3$ $x = t^4$ $y = t^2$

$\frac{dz}{dt} = \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dt}$

$= 6xy^3(4t^3) + 9x^2y^2(2t)$

(b) $z = \cos x \sin y$ $x = u - v$ $y = u^2 + v^2$

$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial u} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial u} =$

$= -\sin x \sin y \cdot 1 + \cos x \cos y \, (2u)$

$\frac{\partial z}{\partial v} = -\sin x \sin y \, (-1) + \cos x \cos y \, (2v)$