Name SOLUTIONS

PRACTICE
SM223 Performance Opportunity # 1 ~~11 September 2007~~
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Given the vectors **A** = < 2, 1, 0 >, **B** = < 3, -1, 2 >, and **C** = < 4, 0, 3 >

1. Find:
   a. **A·B** $= 2\cdot 3 + 1(-1) + 0(2) = 6 - 1 = 5$

   b. The angle between **A** and **B**.
   $\cos\theta = \frac{A\cdot B}{|A||B|} = \frac{5}{\sqrt{5}\sqrt{14}} \rightarrow \theta = 53.3^\circ$ or .9303 radians

   c. $comp_C B$
   $= \frac{C\cdot B}{|C|} = \frac{18}{\sqrt{25}} = \frac{18}{5} = 3.6$

2. Find:
   a. a vector perpendicular to **B**
   LOTS OF ANSWERS X is such that $\bar{X}\cdot\bar{B} = 0$
   so $<1, 3, 0>$

   b. **A X C** $\begin{matrix} 2 & 1 & 0 \\ 4 & 0 & 3 \end{matrix} \rightarrow < 3, -6, -4 >$

   c. True or False: **| A X B | < | A·B|**
   $|\bar{A}||B|\sin\theta < |A||B|\cos\theta \therefore \tan\theta < 1$ not always true

3. Calculate:

   a. < 1, 0, 0 > X < 0, 1, 0 > $\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{matrix} \rightarrow <0, 0, 1>$ or $i \times j = k$

   b. < 0, 1, 0 > X < 1, 0, 0 > $\begin{matrix} 0 & 1 & 0 \\ 1 & 0 & 0 \end{matrix} \rightarrow <0, 0, -1>$ or $j \times i = -k$

   c. < 1, 0, 0 > X < 0, 0, 1 > $\begin{matrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{matrix} \rightarrow <0, -1, 0>$ or $i \times k = -j$

4. a. Let A = (2, 4, - 1) and B = (5, 0, 7) be two points in 3-space. Find the equation of the line through these 2 points.
   $< 3, -4, 8 >$ $\frac{x-2}{3} = \frac{y-4}{-4} = \frac{z+1}{8}$

   b. A plane goes through the point (2, 4, -1) and is parallel to the plane 5x + 2y – 3z = 6. What is the equation of this plane?
   $5(x-2) + 2(y-4) - 3(z+1) = 0$
   $5x + 2y - 3z = 21$

5. a. What is the direction of the plane defined by $3x - 2y + z = 4$?

$\langle 3, -2, 1 \rangle$

b. Consider the two planes: $P_1: x + 2y + z = 5$ and $P_2: x - 2y + z = 7$.

Which of these two planes is perpendicular to $3x - 2y + z = 4$

P_1 is $\perp$

6. Determine whether the line $x = 3 + 8t$, $y = 4 + 5t$, and $z = -3 - t$ is parallel to the plane defined by $x - 3y + 5z = 12$. If it is not parallel, find the point of intersection of the plane and the line.

If parallel then $\langle 8, 5, -1 \rangle \cdot \langle 1, -3, 5 \rangle = 0$

If $x - 3y + 5z = 12$ $\quad$ $8 - 15 - 5 \neq 0$ $\therefore$ not parallel

$\therefore (3+8t) - 3(4+5t) + 5(-3-t) = 12 \rightarrow t = -3$ so $(3+8(-3), 4+5(-3), -3-(-3))$

$\rightarrow (-21, -11, 0)$

7. Match the equations with the sketches of the 3-D surfaces:

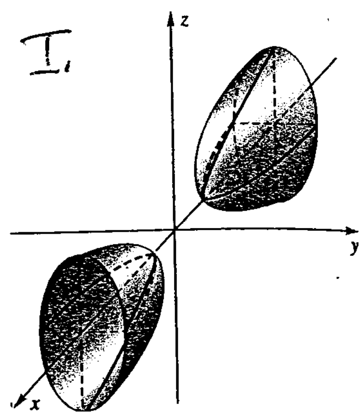
III a. $z = \frac{x^2}{1} + \frac{Y^2}{4}$

II b. $\frac{x^2}{1} + \frac{y^2}{9} + \frac{z^2}{4} = 1$

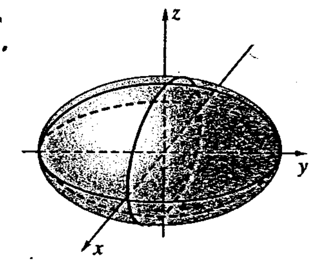
I c. $\frac{x^2}{4} - \frac{y^2}{4} - \frac{z^2}{9} = 1$

IV d. $\frac{x^2}{4} + \frac{y^2}{4} - \frac{z^2}{9} = 1$

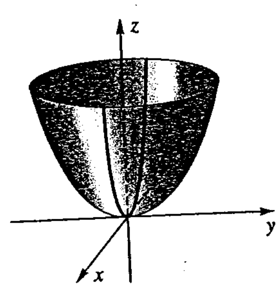
I.



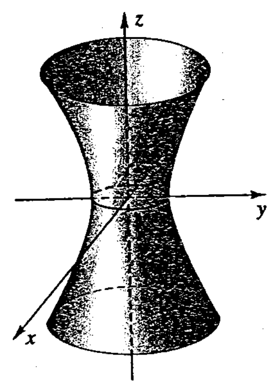
II.



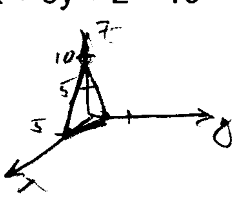
III



IV.

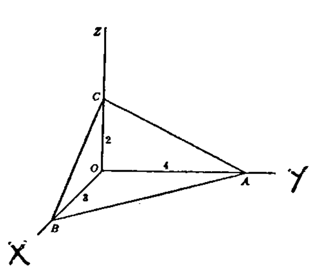


8. a. Sketch $2x + 5y + z = 10$



b. From the sketch below, find the equation of the plane of the form

$ax + by + cz = 12$ (i.e. find the values for a, b, and c).



$by = 12$
$4b = 12 \quad b = 3$

$cz = 12$
$2c = 12$
$c = 6$

$ax = 12$
$3a = 12$
$a = \frac{12}{3} = 4$

$4x + 3y + 6z = 12$