SM223 Practice Performance Opportunity 3A 27 Oct 08

1. Consider the function $x^2 + 2y^2 - z^2 = 1$. Using various values of z, sketch a contour plot of this function.

2. Let $z = 8 - x^2 - 2y$. Draw a sketch of the graph of z and a contour plot of z showing the level curves at z = 10, 8, 6, 4, 2 and 0.

3. If $w = f(x, y, z) = x^2y + yz^2 + z^3$, verify that $xf_x + yf_y + zf_z = 3f(x, y, z)$.

4. If $z = \frac{1}{2}\ln(x^2 + y^2)$ and $x = r\,e^s$ and $y = r\,e^{-s}$ find $\frac{\partial z}{\partial r}$ and $\frac{\partial z}{\partial s}$.

5. If $f(x, y) = e^x \sin(y) + \ln(x\,y)$, find a.) $\frac{\partial z}{\partial x}$ b. $\frac{\partial^2 z}{\partial x^2}$ c. $\frac{\partial^2 z}{\partial y\,\partial x}$

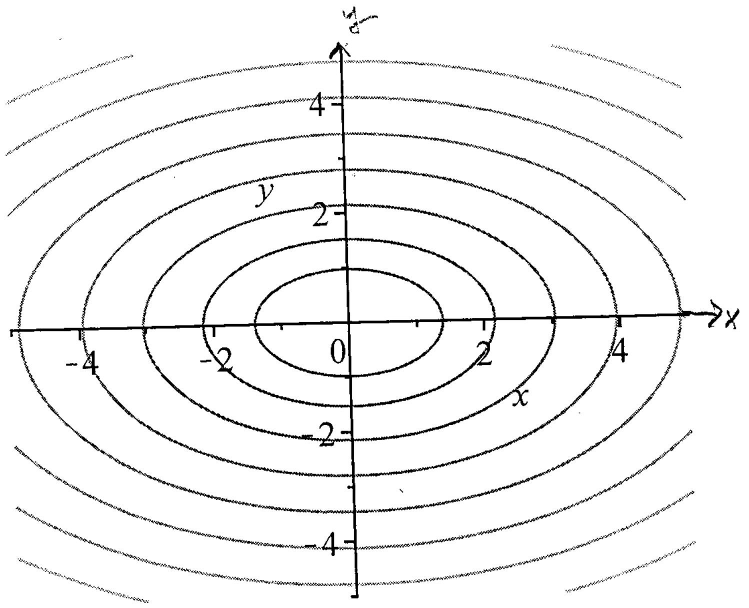
6. If $w = f(x, y, z) = 3x^2 + xy - 2y^2 - yz + z^2$, find the directional derivative at the point (1, - 2, - 1) in the direction $< 2, -2, -1 >$.

7. Find the equation of the tangent plane and normal line to the function $f(x, y, z) = 4x^2 + y^2 - 16z = 0$ at the point (2, 4, 2).

8. Find symmetric equations of the tangent line to the curve of intersection of the surfaces $3x^2 + 2y^2 + z^2 = 49$ and $x^2 + y^2 - 2z^2 = 10$ at the point (3, -3, 2).

SOLUTIONS   PPU3A   SM 223   28 OCT 08

1.

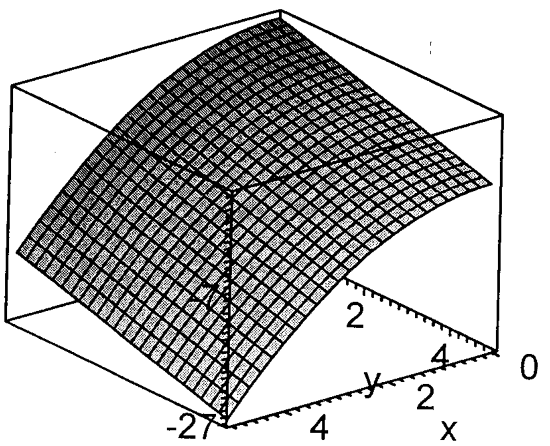


CONTOUR PLOT

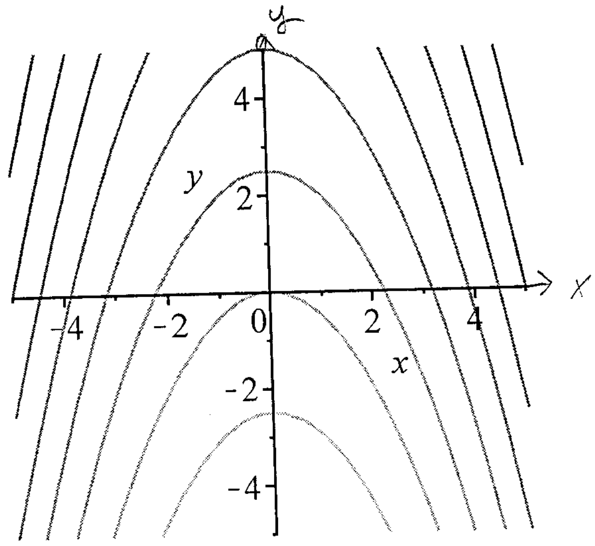
$x^2+2y^2-z^2=1$

2. $z = 8 - x^2 - 2y$

SKETCH



CONTOUR PLOT



3. $w = f(x,y,z) = x^2y + yz^2 + z^3$ VERIFY: $x f_x + y f_y + z f_z = 3f(x,y,z)$

$f_x = 2xy \quad f_y = x^2+z^2 \quad f_z = 2yz + 3z^2$

$x(2xy) + y(x^2+z^2) + z(2yz+3z^2)$

$2x^2y + yx^2 + yz^2 + 2yz^2 + 3z^3 = 3x^2y + 3yz^2 + 3z^3 = 3(x^2y + yz^2 + z^3)$

QED

4. $z = \frac{\ln(x^2+y^2)}{2} \quad x = re^s \quad y = re^{-s}$

$\frac{\partial z}{\partial r} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial r} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial r} = \frac{x}{x^2+y^2}\cdot e^s + \frac{y}{x^2+y^2}e^{-s}$

$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial s} = \frac{x}{x^2+y^2}\cdot re^s + \frac{y}{x^2+y^2}(-re^{-s})$

5. $z=f(x,y) = e^x \sin y + \ln(xy)$

ⓐ $\frac{\partial z}{\partial x} = e^x \sin y + \frac{1}{xy}\cdot y = e^x \sin y + \frac{1}{x}$

ⓑ $\frac{\partial^2 z}{\partial x^2} = e^x \sin y - \frac{1}{x^2}$

ⓒ $\frac{\partial^2 z}{\partial y \partial x} = e^x \cos y$

6. $w = f(x,y,z) = 3x^2 + xy - 2y^2 - yz + z^2$ @ $(1,-2,-1)$

DIR. DER $\rightarrow \langle 2,-2,-1\rangle \rightarrow \langle \frac{2}{3}, -\frac{2}{3}, -\frac{1}{3}\rangle$ unit vector

$\nabla w = \langle 6x+y,\ x-4y-z,\ -y+2z\rangle$

$(1,-2,-1)$ $\langle 4, 10, 0\rangle$ $\langle 4,10,0\rangle \cdot \langle \frac{2}{3}, -\frac{2}{3}, -\frac{1}{3}\rangle$

$= \frac{8}{3} - \frac{20}{3} + 0 = -12/3 = -4$

7. $f(x,y,z) = 4x^2 + y^2 - 16z = 0$ $\nabla f = \langle 8x, 2y, -16\rangle$

tangent plane: @ $(2,4,2)$ $\nabla f = \langle 16, 8, -16\rangle$

$2(x-2) + 1(y-4) - 2(z-2) = 0$ $\approx \langle 2,1,-2\rangle$

NORMAL LINE

$\frac{x-2}{2} = \frac{y-4}{1} = \frac{z-2}{-2}$

8. $3x^2 + 2y^2 + z^2 - 49 = 0$ $x^2 + y^2 - 2z^2 - 10 = 0$

$\nabla_1 = \langle 6x, 4y, 2z\rangle$ @ $(3,-3,2)$ $\nabla_2 = \langle 2x, 2y, -4z\rangle$

$\nabla_1 = \langle 18, -12, 4\rangle$ $\nabla_2 = \langle 6, -6, -8\rangle$

$\approx \langle 9, -6, 2\rangle$ $\approx \langle 3, -3, -4\rangle$

$\nabla_1 \times \nabla_2 = \begin{vmatrix} i & j & k \\ 9 & -6 & 2 \\ 3 & -3 & -4 \end{vmatrix} = \langle 30, 42, -9\rangle$

$\therefore \frac{x-3}{30} = \frac{y+3}{42} = \frac{z-2}{-9}$