SM223 28OCT08

PRACTICE P03 SOLUTIONS

$\boxed{1.}$ $f(x,y,z) = \ln(x^2+y^2) - z = 0$

(a) TANGENT PLANE EQUATION $f_x = \dfrac{2x}{x^2+y^2}$; $f_y = \dfrac{2y}{x^2+y^2}$, $f_z = -1$

@ $(-2, 1, \ln 5)$ $f_x = \dfrac{-4}{5}$, $f_y = \dfrac{2}{5}$, $f_z = -1$

so $-\frac{4}{5}(x+2) + \frac{2}{5}(y-1) - 1(z - \ln 5) = 0$

(b) $\dfrac{x+2}{-4/5} = \dfrac{y-1}{2/5} = \dfrac{(z-\ln 5)}{-1}$

$\boxed{2}$ $T(x,y) = e^x \cos y + e^y \cos x$

$\dfrac{\partial T}{\partial x} = e^x \cos y - e^y \sin x$ $\dfrac{\partial T}{\partial y} = -e^x \sin y + e^y \cos x$

(a) $\nabla T = \langle \frac{\partial T}{\partial x}, \frac{\partial T}{\partial y} \rangle$ (b) at $(0,0)$ $\nabla T_{(0,0)} = \langle e^0 \cos 0 - e^0 \sin 0, -e^0 \sin 0 + e^0 \cos 0 \rangle$

(c) $|\nabla T| = \sqrt{1^2+1^2} = \sqrt{2}$ $= \langle 1, 1 \rangle$ In this direction for most rapid increase

$\boxed{3.}$ $f(x,y) = \dfrac{2x}{x-y}$ at $(1,0)$ DIRECTIONA DERIVATIVE IN DIRECTION $\langle 1, -\sqrt{3} \rangle$

$\nabla f \cdot \hat{u}$ = DIR DER $\nabla f = \left\langle \dfrac{2(x-y) - 2x}{(x-y)^2}, \dfrac{+2x}{(x-y)^2} \right\rangle = \langle 0, 2 \rangle$

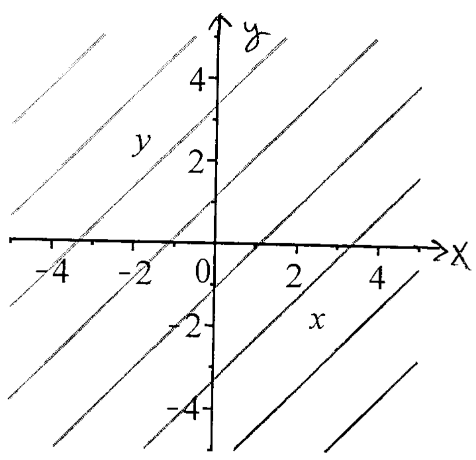
$\langle 0, 2 \rangle \cdot \underbrace{\langle \tfrac{1}{2}, \tfrac{-\sqrt{3}}{2} \rangle}_{\text{UNIT VECTOR}} = -\sqrt{3}$

$\boxed{4.}$ $z = \dfrac{x^2}{2} - y^2$ at point $(-1, 1, -\tfrac{1}{2})$

$\nabla z = \langle x, -2y \rangle \longrightarrow \langle -1, -2 \rangle$

STEEPEST ASCENT IS ∇z

$\boxed{5.}$ CONTOUR PLOT $f(x,y) = y - x$



$\boxed{6.}$

ⓐ $\underset{(0,0)}{f_x} = \frac{\Delta z}{\Delta x} = \frac{2.2-1.8}{.1-(-.1)} = \frac{.4}{.2} = 2$

$\underset{(0,0)}{f_y} = \frac{\Delta z}{\Delta y} = \frac{1.6-2.4}{.2-(-.2)} = \frac{-.8}{.4} = -2$

ⓑ $\nabla f = \langle 2, -2 \rangle$

DIRECTION $= \langle 1, 1 \rangle \Rightarrow \langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$ unit vector

$\therefore \langle 2, -2 \rangle \cdot \langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle = 0$

$\boxed{7.}$

A - V

B - II

C - IV

D - III

E - VI

F - I