SM223 Practice Performance Opportunity # 4

1. Locate and classify all critical points for $z = f(x, y) = 2x^2 - y^3 - 2xy$.

2. Locate and classify all critical points for $z = f(x, y) = x^3 - 2y^2 - 2y^4 + 3x^2y$.

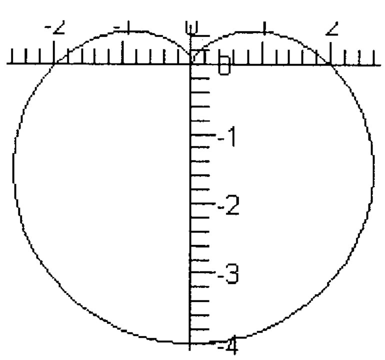
3. Suppose that the temperature of a metal plate is given by $T(x, y) = x^2 + 2x + y^2$ for points on the elliptical plate defined by $x^2 + 4y^2 \le 24$. Find the maximum and minimum temperatures on the plate using LaGrange multiplier techniques.

4. Compute a Riemann Sum for $f(x, y) = 5x - 2y$ with 4 equal-sized rectangles over the region from x = 1 to x = 3 and y = 0 to y = 1, evaluating the function at the mid point of each rectangle.

5. Let R be the region bounded by the graphs of y = x, y = 0, and x = 4. Set up and evaluate the integral $\iint_R (4e^{x^2} - 5\sin y)\,dA$.

6. Reverse the order of integration of the iterated integral $\int_0^1 dy \int_{x=y}^{x=1} e^{x^2}\,dx$ and evaluate both of these iterated integrals.

7. Use polar coordinates to evaluate the iterated integral $\iint_R dA$ over the region defined by $r = 2 - 2\sin\theta$, sketched below.



SOLUTIONS TO PP04 26NOV07

1. $f(x,y) = 2x^2 - y^3 - 2xy$

$f_x = 4x - 2y = 0$

$f_y = -3y^2 - 2x = 0$

solve($4x-2y=0$ AND $-3y^2-2x=0$, {x, y})

$x = -1/6 \quad y = -1/3$

$x = 0 \quad y = 0$

Points: $(0,0)$ $(-1/6, -1/3)$

$f_{xx} = 4 \quad f_{yy} = -6y$

$f_{xy} = -2 \quad f_{yx} = -2$

$$D = \begin{vmatrix} 4 & -2 \\ -2 & -6y \end{vmatrix}$$

@ $(0,0)$ $D = -4$ saddle point

@ $(-1/6, -1/3)$ $D = 4+2 = 6$

$f_{xx} > 0 \therefore$ minimum

2. $f(x,y) = x^3 - 2y^2 - 2y^4 + 3x^2y$

$f_x = 3x^2 + 6xy = 0$

$f_y = -4y - 8y^3 + 3x^2 = 0$

$x=-2, y=1$ or $x=-1, y=1/2$ or $x=0, y=0$

$f_{xx} = 6x + 6y$

$f_{yy} = -4 - 24y^2$

$f_{xy} = 6x = f_{yx}$

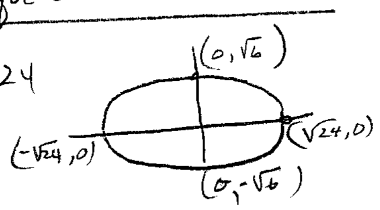
$$D = \begin{vmatrix} 6x+6y & 6x \\ 6x & -4-24y^2 \end{vmatrix}$$

@ $(-2,1)$ $D = 24 > 0$, $f_{xx} < 0 \therefore$ max

@ $(-1, 1/2)$ $D < 0$ saddle point

@ $(0,0)$ no conclusion for $D = 0$

3. $T(x,y) = x^2 + 2x + y^2 \qquad x^2 + 4y^2 \le 24$



$T_x = 2x + 2 = 0 \quad T_{xx} = 2 \quad T_{xy} = 0$

$T_y = 2y = 0 \quad T_{yy} = 2$

$y = 0 \quad x = -1$

$(-1, 0)$

$$D = \begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} = 4$$

$T_{xx} > 0 \therefore$ minimum

$\boxed{T(-1,0) = -1}$

$g(x,y) = x^2 + 4y^2 - 24$

$\nabla T = \lambda \nabla g(x,y) \qquad \langle 2x+2, 2y \rangle = \lambda \langle 2x, 8y \rangle$

$2x + 2 = 2x\lambda \qquad 2y = 8y\lambda$ or

$x = -2\sqrt{6} \quad y = 0 \quad \lambda = \frac{-(\sqrt{6}-12)}{12}$

$x = -4/3 \quad y = \frac{-5\sqrt{2}}{3} \quad \lambda = 1/4$

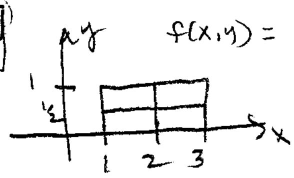
$x = -4/3 \quad y = \frac{5\sqrt{2}}{3} \quad \lambda = 1/4$

$x = 2\sqrt{6} \quad y = 0 \quad \lambda = \frac{\sqrt{6}+12}{12}$

	T
$(-\sqrt{24}, 0)$	33.8
$(-4/3, \frac{\sqrt{50}}{3})$	4.7
$(-4/3, -\frac{\sqrt{50}}{3})$	4.7
$(\sqrt{24}, 0)$	14.2

$\therefore$ minimum @ $(-1,0) = -1$ on the interior

4. $f(x,y) = 5x - 2y$

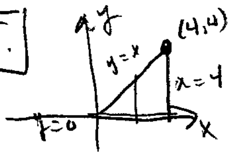


$R = \frac{1}{2}\left[f(\tfrac{3}{2},\tfrac{1}{4}) + f(\tfrac{5}{2},\tfrac{1}{4}) + f(\tfrac{3}{2},\tfrac{3}{4}) + f(\tfrac{5}{2},\tfrac{3}{4})\right]$

$= \frac{1}{2}\left[\frac{15}{2} - \frac{1}{2} + \frac{25}{2} - \frac{1}{2} + \frac{15}{2} - \frac{3}{2} + \frac{25}{2} - \frac{3}{2}\right]$

$= \frac{1}{2}[40 - 1 - 3] = \frac{1}{2}[36] = \underline{18}$

5.

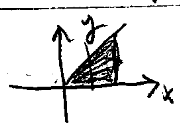


$y = x$
$y = 0$
$x = 4$

$$\int_0^4 dx \int_{y=0}^{y=x} \left(4e^{x^2} - 5\sin y\right) dy = 1.78 * 10^7$$

= ∫(∫(4*e^(x^2) - 5*sin(y), y, 0, x), x, 0, 4) =

6. $\displaystyle\int_0^1 dy \int_{x=y}^{x=1} e^{x^2} dx$



$\displaystyle\rightarrow \int_{x=0}^{x=1} dx \int_{y=0}^{y=x} e^{x^2} dy = .859141$

7. $\displaystyle\int_0^{2\pi} d\theta \int_{r=0}^{r=2-2\sin\theta} r\, dr = 18.8496$