SOLUTIONS

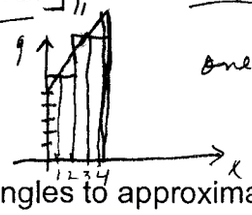
SM122 Practice Performance Opportunity # 1 22 Jan 2009

1. If $f'(x)=\frac{1}{x}+x^2$ and $f(1)=\frac{2}{3}$

a. Find f(x). $= \int\left(\frac{1}{x}+x^2\right)dx = \ln x + \frac{x^3}{3} + C$ @ $x=1$ $\frac{2}{3} = \ln 1 + \frac{1}{3} + C$ So $C = \frac{1}{3}$ and $f(x) = \ln x + \frac{x^3}{3} + \frac{1}{3}$

b. Evaluate: $\int_1^e \frac{\ln(x)}{x}\,dx = \left[\frac{(\ln x)^2}{2}\right]_1^e$ $\frac{(\ln e)^2 - (\ln 1)^2}{2} = \frac{1^2 - 0}{2} = \frac{1}{2}$

2. Let $y=2x+5$ with $0 \le x \le 4$.



one possibility:
$f(1)\cdot 2 + f(3)\cdot 2$
$7\cdot 2 + 11\cdot 2 = 14+22 = 36$

Use a Riemann mid sum with 2 rectangles to approximate the area in the first quadrant of this region.

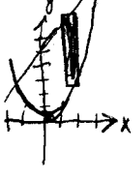
3. Evaluate:

a. $\int_0^0 (x^7+x^6+x^5+x^4+x^3+x^2+x+1)(7x^6+6x^5+5x^4+4x^3+3x^2+2x+1)\,dx = 0$

b. Use Human Intervention to find: $\int_0^2 x\sqrt{1+x^2}\,dx$. Show your work for credit.

$\frac{1}{2}\int_0^2 2x(1+x^2)^{1/2}\,dx = \frac{2}{3}\cdot\frac{1}{2}(1+x^2)^{3/2}\Big|_0^2 = \frac{1}{3}(\sqrt{5}^3 - 1) = \frac{10.18}{3} \approx 3.4$

4. Consider the region defined by the functions $y=f(x)=x^2$ and $y=g(x)=x+6$.



a. Find the area of the region bounded by these functions. $\int_{-2}^{3}[(x+6)-x^2]\,dx = 20.83$

b. Set up the integral to find the volume when the region is rotated about the x-axis. $\int_{-2}^{3}[\pi(x+6)^2 - \pi(x^2)^2]\,dx$

5. Rotate the region defined in problem #4 about the line y= -1 and find its volume. $\int_{-2}^{3}\pi(x+6-(-1))^2 - \pi(x^2-(-1))^2\,dx$

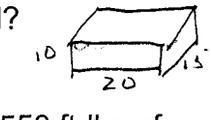
6. a. How much work is done if a 2.2 pound brick is raised 5 feet? $W = 2.2\cdot 5 = 11$

b. How much work is done if a 1 kg brick is raised 1.521 meters? $W = 1(9.8)(1.521) = 14.9$

c. If 1 kg = 2.2 pounds and 1.521 meters = 5 feet, how many joules are in a foot-pound?
11 ft lbs $= 14.9$ J $\therefore$ 1 ft lb $= 14.9/11 = 1.35$ J

7. A swimming pool is built in the shape of a rectangular tank, 10 ft deep, 15 ft wide and 20 ft long.

a. If the pool is filled to 1 ft below the top, how much work is required to pump all of the water into a drain at the top edge of the pool?



$\int_1^{10} x\,(62.5)(20)(15)\,dx = 928{,}125$ ft lbs

b. A one-horsepower motor can do 550 ft-lbs of work per second. What size motor is required to empty the pool in one hour? $928125/550 = 1687.5$ sec $\div 3600 \rightarrow .47$ HP or about a half a horse.

8. If the average value of the depth of a bay were 12 feet, could you navigate your ship, with a 7-foot draft, up the bay? Please explain in a paragraph of lucid English prose.

NOT NECESSARILY, UNLESS THE SHIP STAYED IN A DREDGED CHANNEL WITH A DEPTH GREATER THAN 7'. THERE COULD BE DEEP HOLES SURROUNDED BY SHALLOW POOL ALL OVER THE BAY AND AVERAGING 7'.