Significant Figures and Rounding – Explanations and Examples

Read pages 18-22 in your Lab Manual for a more thorough discussion of the meaning of significant figures and how it relates to accuracy, precision, and error.

1. Why do we have to worry about significant figures anyway?

All measurements are inexact – they will have some error or uncertainty associated with them – because our measuring techniques are not exact. It is common practice for scientists to report measurements with enough decimal places so that only the last one has any uncertainty. For example, a measurement reported as 45.67 mL indicates that the piece of equipment being used could be measured precisely to 0.1 mL and a reasonable guess can be made about the hundredths place. So, this measurement is taken as being somewhere between 45.66 and 45.68 mL. There is some uncertainty in that last decimal place, but since a reasonable estimate can be made for its value, the 7 is still significant. Using the same piece of equipment, it would be dishonest to report a measurement of 32.446 mL because the equipment can only measure out to ± 0.01 mL. Any decimal places beyond the hundredths place would be pure guesses, and not at all significant for the value of this measurement.

So, significant figures are important for establishing the precision of measurements and any values calculated from those measurements.

2. How can you tell how many significant figures a particular number has?

Rule:		Examples:
(1)	All nonzero digits ARE significant.	356 has 3 SF 2.4553 has 5 SF
(2)	Zeros between nonzero digits ARE significant.	2305 has 4 SF 7.0095 has 5 SF
(3)	Zeros at the end of a number that contains a decimal point ARE significant – they are measured values.	244.50 has 5 SF 1.20 has 3 SF 70.00 has 4 SF
(4)	Zeros to the left of a nonzero digit ARE NOT significant – they are not measured values, they only show the position of the decimal point.	0.0033 has 2 SF 0.7780 has 4 SF 0.005 has 1 SF
(5)	Zeros at the end of a number that does not contain a decimal point MAY OR MAY NOT BE significant – use scientific notation to be clear.	1500 has 2 or 3 or 4 SF 1.5×10^3 has 2 SF 1.50×10^3 has 3 SF 1.500×10^3 has 4 SF
(6)	Exact numbers (which are numbers in definitions or counting numbers) can be treated as if they have an infinite number of significant figures.	12 inches in a foot 37 cars 1948 atoms 2.54 cm in 1 inch

3. Aren't all whole numbers exact?

NO. A whole number is one with no digits after the decimal point, such as 334 or 12. These numbers are only exact in certain situations or contexts. For example, 12 is exact in the context of 12 inches to a foot. Also, 12 is exact in the context of 12 eggs in a dozen. Also, 12 is exact in the context of 12 dogs in this house. However, 12 is not exact in the context of a measurement only precise to ± 1 unit, such as a graduated cylinder that can only be read to the nearest mL. Then, the measurement would be 12 mL, meaning 11-13 mL and is not exact.

4. When you are rounding off numbers, how do you know whether to round up or round down? And what exactly does "round up" or "round down" mean anyway?

Method A

Rule – Look at the leftmost digit to be dropped. If it is…	
(1)	…less than 5, then the preceding number is unchanged (round down)
(2)	…more than 5, then increase the preceding number by one (round up)
(3)	…5 with other nonzero digits after it, then increase the preceding number by one (round up)
(4)	…5 with no nonzero digits after it, then either increase or decrease the preceding number by one (round up or round down) depending on which one gives you an even number for the last digit

Method B

Rule…	
(1)	Determine which two numbers you are deciding between, and choose the one which is closer to the original, unrounded number.
(2)	If the number to be rounded is exactly half-way between two numbers, either round up or down depending on which one gives you an even number for the last digit.

Note: In this course, we are currently using the even/odd rule which is one way to handle numbers that are exactly half-way between the two choices – there are other, equally valid ways to do this.

EXAMPLES: round each of the following numbers to 3 significant figures

	METHOD A		METHOD B	
number to be rounded	**leftmost digit to be dropped**	**answer (rule)**	**numbers to choose between**	**answer (rule)**
1.0623	2	1.06 (1)	1.06 and 1.07	1.06 (1)
1.0481	8	1.05 (2)	1.04 and 1.05	1.05 (1)
1.0532	3	1.05 (1)	1.05 and 1.06	1.05 (1)
1.0550	5	1.06 (4)	1.05 and 1.06	1.06 (2)
1.0257	5	1.03 (3)	1.02 and 1.03	1.03 (1)
17.050	5	17.0 (4)	17.0 and 17.1	17.0 (2)
189.39	3	189 (1)	189 and 190	189 (1)

5. What happens if you have to round off a really big number?

You still have to keep the size (magnitude) of the number intact. For example, if you have to round off 5,345,666 to two significant figures, you cannot just drop the last digits to give 53. That would dramatically change the size of the number (from about 5 million to about 50). Instead, you put zeros in the places of the digits you drop. This example would be 5,300,000. However, the number of significant figures in this new number is ambiguous. Therefore, switch to scientific notation to be clear: 5.3×10^{6}. It might be easier to change the number to scientific notation first, then round it off.

6. **How do you decide how many significant figures your answer should have when you are adding or subtracting numbers?**

When adding or subtracting, the answer should have as many DECIMAL PLACES as the fewest number of DECIMAL PLACES in the numbers you are adding and subtracting. Decimal places are the number of digits after the decimal point.

Example	Answer – not rounded off	Answer – rounded off
90.00 cm + 3.456 cm + 12.3 cm	105.756 cm	105.8 cm (1 decimal place)
0.0248 g + 0.00478 g – 0.045 g	–0.01542 g	–0.015 g (3 decimal places)
456 mL + 0.332 mL	456.332 mL	456 mL (no decimal places)
$1.00\times10^{-2} - 9.786\times10^{-4}$	9.0214×10^{-3}	9.0×10^{-3} or 0.0090 (see below)

When adding or subtracting numbers in scientific notation, you must compare the decimal places for the numbers <u>out of</u> scientific notation, or, alternatively, change the numbers so that they all have the <u>same exponent</u>, then compare the decimal places. Make sure that if your final answer is given in scientific notation, that you use standard scientific notation (just one digit in front of the decimal point). The last example above then becomes…

$0.0100 - 0.0009786 = 0.0090214$ or 0.0090 (4 decimal places) or 9.0×10^{-3}

or $1.00\text{x}10^{-2} - 0.09786\text{x}10^{-2} = 0.90\text{x}10^{-2}$ (2 decimal places) = 0.0090 or 9.0×10^{-3}

or $100\text{x}10^{-4} - 9.786\text{x}10^{-4} = 90\text{x}10^{-4}$ (no decimal places) = 0.0090 or 9.0×10^{-3}

7. **How do you decide how many significant figures your answer should have when you are multiplying or dividing numbers?**

When multiplying or dividing, the answer should have as many TOTAL SIGNIFICANT FIGURES as the fewest number of TOTAL SIGNIFICANT FIGURES in the numbers you are multiplying and dividing.

Example	Answer – not rounded off	Answer – rounded off
235 miles / 3.5 hours	67.14285714 miles/hour	67 miles/hour (2 SF)
16 cm × 35.66 cm × 2.31 cm	1317.9936 cm^3	1.3×10^{3} cm^3 (2 SF)
(3.447×10^{-4} kg) × (5.51×10^{3} m/s)	1.899297 kg·m/s	1.90 kg·m/s (3 SF)

8. **What if you do a calculation where you have adding or subtracting AND multiplying or dividing?**

In this case, you apply each rounding rule as you do the individual calculations.

- For example:

$$\frac{257.665\,g - 249.457\,g}{23.6\,mL} = \frac{8.208\,g}{23.6\,mL} = 0.34779661\ g/mL = 0.348\ g/mL$$

  - In the subtraction step, the answer should be rounded to 3 decimal places. This will give that number a total of 4 significant figures: 8.208.
  - In the next step, this number with 4 SF is divided by a number with 3 SF. The answer needs to be rounded off to 3 SF: 0.348 g/mL

- Here's another example:

(34 mL x 0.847 g/mL) + (105 mL x 1.01 g/mL) = 29 g (2 SF) + 106 g (3 SF) = 135 g (no decimal places)

- And one more example:

$$\text{average mass} = \frac{3.654\,g + 3.598\,g + 3.671\,g}{3} = \frac{10.923\,g\ (3\ \text{decimal places})}{3} = 3.6410\ g\ (5\ SF)$$

9. How do you figure out what units to put for the answer?

Here are some guidelines for determining the correct units for the answers:

Situation	Example
When two numbers with units are multiplied together, the units are also multiplied	$2\ cm \times 4\ cm = 8\ cm^2$
When two numbers with the same unit are divided, the units cancel out.	$\frac{82\,g}{2.0\,g} = 42$
When two numbers with different units are divided, both units are in the final answer, one in the numerator and one in the denominator.	$\frac{16\,g}{4.0\,mL} = 4.0\ \frac{g}{mL}$
For multiplication problems with multiple units involved, cancel out any units that appear in both the numerator and denominator of the overall problem.	$48\ in \times \left(\frac{1.0\,ft}{12\,in}\right) = 4.0\ ft$
When two numbers with units are added or subtracted, the answer has the same unit as each of the numbers involved.	23.0 cm + 16.0 cm = 39.0 cm