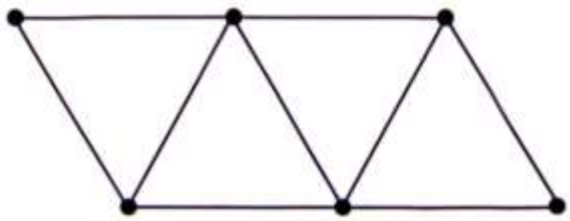
Squeezing an Euler Circuit on the Original Graph

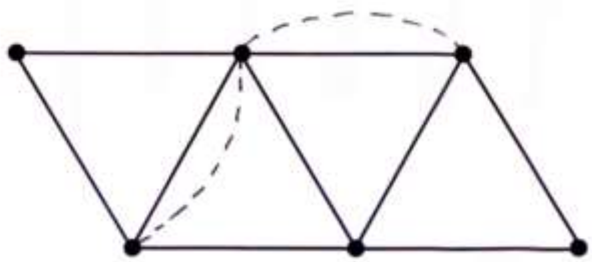
An original graph may or may not have Euler circuits. An Eulerized graph always has Euler circuits. Once the graph has been Eulerized, a best Euler circuit is marked on it. Then this best Euler circuit is marked on the original graph, causing some backtracking if a person traveled following it. This action of putting the Euler circuit back on the given graph is called a **squeeze**. It results in a plan on how the person/vehicle can travel servicing the neighborhood, using edges repeatedly if necessary (but only if necessary) so that all streets have been included in the circuit.

Example 10

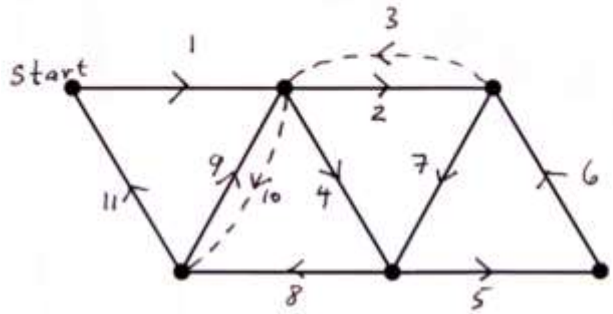
Given original graph:



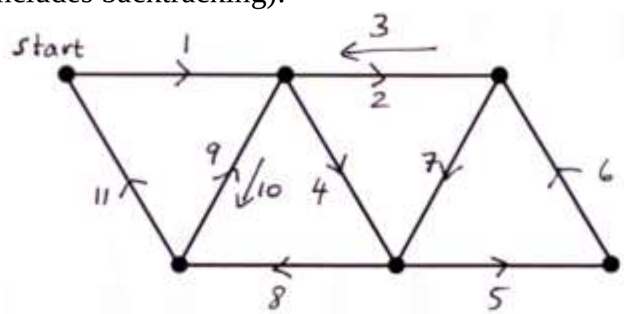
Eulerized graph (note: other Eulerizations are possible):



Eulerized graph from above with one Euler circuit marked (note: other Euler circuits are possible for this Eulerization):



Squeeze (includes backtracking):



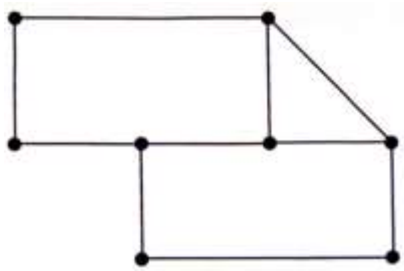
The squeeze will be used as a guide on how to travel on the graph in an efficient circuit.

Author of pages 11-15: Prof. Sybille Clayton, Anne Arundel Community College

Note: The location of the duplicated edges put during an Eulerization as well as the Euler circuit chosen on the Eulerized graph can vary. To ensure a best solution, backtracking has to be kept at a minimum.

Try it Now # 4

1. Find an Eulerization for the given graph. Then mark an Euler circuit on the Eulerized graph and squeeze it on the original graph.



(*Answers on top of next page*.)

Putting It All Together – Best Routes Through a Neighborhood

Applications such as garbage pick-up, mail delivery, snow removal, pothole inspections, etc. require travel through all streets of a neighborhood (often entering and exiting at a specific entry point). In some of these scenarios the service person has to go down each street only once, but in some instances the right and left side of the street have to be traveled separately (e.g. a mail-carrier needs to deliver mail to all mailboxes, even if they are on opposite sides of the street).
The original graph drawn is the model of the neighborhood but also the real-word scenario and needs to take this into consideration.

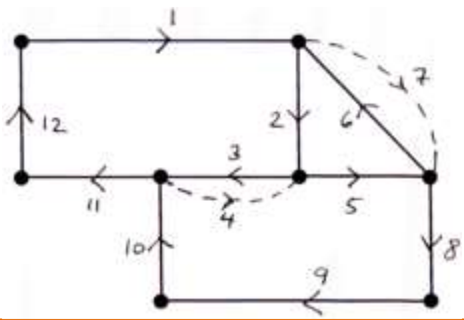
Note: Double edges (two edges connecting the same pair of vertices) may already exist in an original graph – e.g. in a mail-carrier scenario where both sides of the street are traveled independently. Duplicating one of these edges will then result in a third connection between this pair of vertices and mean that one of the sides of this streets (either the right or the left side) will be traveled twice while the other side is only traveled once.

Real world scenarios may be even more complicated than what we can easily discuss on the model (e.g. an additional restriction that a mail-carrier is not allowed to drive against traffic exists), but we will not go into such details here. Once we understand how a best way to travel through a neighborhood is created the ideas can be adapted to cover restrictions like these as well.

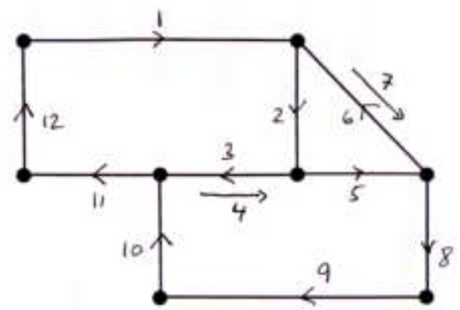
Let's look at one example of a garbage pickup in one neighborhood, followed by mail-delivery in another.

ANSWERS to Try it Now # 4

1. Eulerized graph with Euler circuit marked:

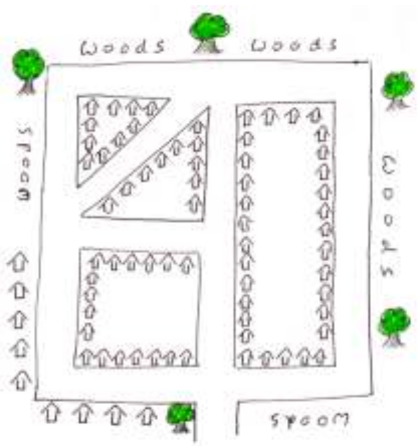


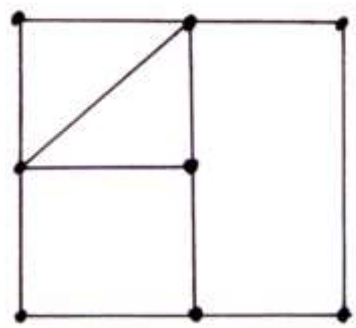
Squeeze:



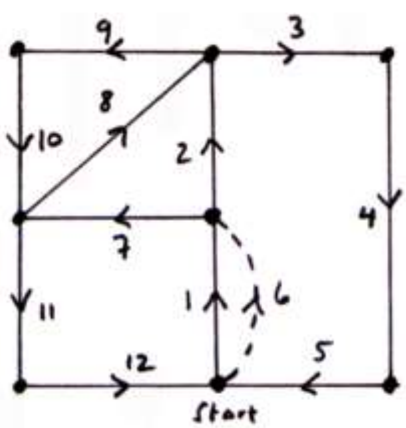
Example 11

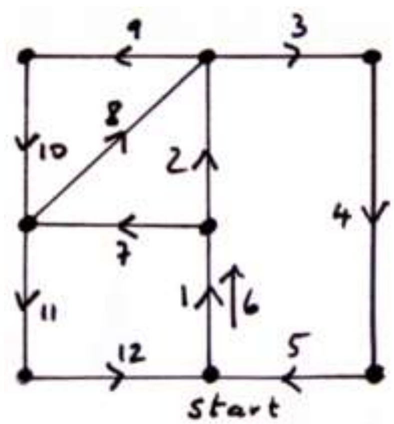
Shown below is a neighborhood in which garbage is picked up by a garbage truck. Each house has a garbage can out on the curbside that needs to be emptied. On the right, you see the graph that represents the neighborhood.





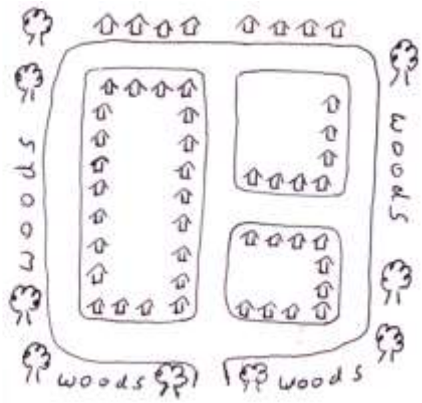
The left picture shows an Eulerization and a marked Euler circuit with start and stop at the entry point to the neighborhood. The right picture shows the squeeze onto the original graph.



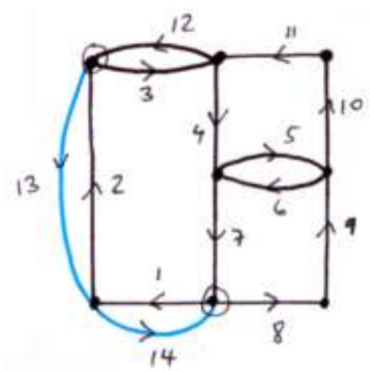


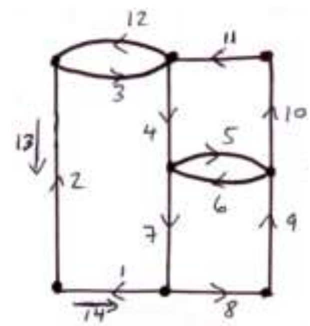
Example 12

Shown below is a neighborhood in which a mail-carrier delivers mail to houses. We are assuming that the mailboxes are located in front of the houses on the street.
On the right, you see the graph that represents the neighborhood and shows each side of the street that has mailboxes separately.



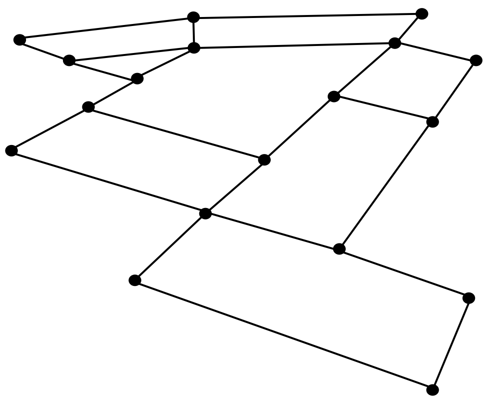
The left picture shows an Eulerization and a marked Euler circuit with start and stop at the entry point to the neighborhood. The right picture shows the squeeze onto the original graph.

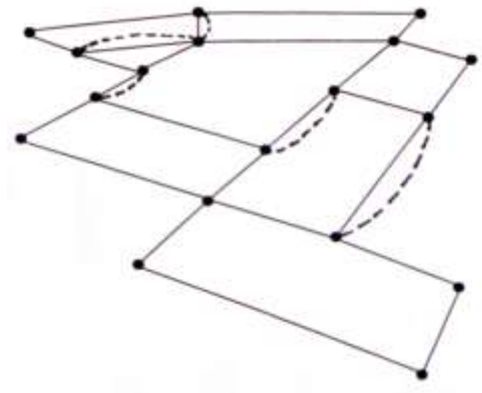




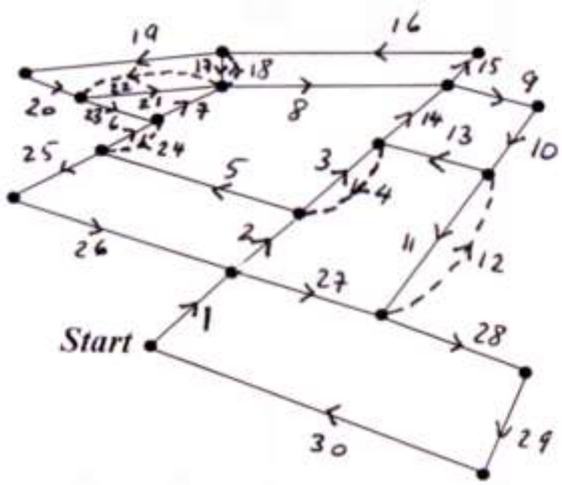
Example 13

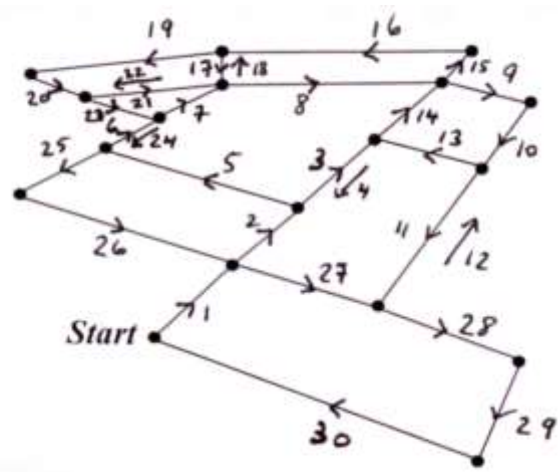
And as a last example performing an Eulerization, the neighborhood from the beginning of the graph theory introduction is revisited.
The left picture shows the graph modeled after the neighborhood that will help the lawn inspector create an efficient route through the neighborhood. The right picture shows the Eulerized graph.





Below, the left picture has an Euler circuit marked on the graph, the right picture shows the squeeze.





Now the lawn inspector can follow the route shown in the squeeze, which will start and stop at the intersection where she may have parked her car.

Eulerizing a graph and finding (and squeezing) an Euler circuit can solve many application problems in which each edge of a graph that models the real-world situation has to be traveled in its entirety. The given examples were just a small selection of where this theory can be applied.