SOLUTIONS

MAT 012 **Lecture Notes: ch 12.1, p.713, The Algebra of Functions**

Definitions:

$$(f+g)(x) = f(x)+g(x)$$
$$(f-g)(x) = f(x)-g(x)$$
$$(fg)(x) = f(x)\cdot g(x)$$
$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, \text{ for } g(x)\neq 0$$

Example: Given $f(x)=2x^2-x$ and $g(x)=4x-3$

Evaluate $(f-g)(-1)$ in two different ways:

1st way to find $(f-g)(-1)$: Use the definition and evaluate each piece separately.

$(f-g)(-1) = f(-1)-g(-1) = [2(-1)^2-(-1)]-[4(-1)-3]$

$= (2\cdot 1+1)-(-4-3) = (2+1)-(-7) = 3+7 = 10$

2nd way to find $(f-g)(-1)$:

Find $(f-g)(x)$, then substitute $x=-1$ into this function.

$(f-g)(x) = f(x)-g(x) = (2x^2-x)-(4x-3) = 2x^2-x-4x+3$

$= 2x^2-5x+3$

$(f-g)(-1) = 2(-1)^2-5(-1)+3 = 2\cdot 1+5+3 = 2+5+3 = 10$

Example: Given $f(x)=x^2-3$ and $g(x)=4x^2-2x$. Give and simplify: $(f+g)(x)$

$(f+g)(x) = f(x)+g(x) = (x^2-3)+(4x^2-2x) = x^2-3+4x^2-2x$

$= 5x^2-2x-3$

Example: Given $f(x)=2x+1$ and $g(x)=3x-4$. Give and simplify: $(fg)(x)$

$(fg)(x) = f(x)\cdot g(x) = (2x+1)(3x-4)$

$= 6x^2-8x+3x-4$

$= 6x^2-5x-4$

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Example: Given $f(x)=5x^2$ and $g(x)=3x+6$.

Give $\left(\frac{f}{g}\right)(x)$, factor the denominator and give the function's domain:

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{5x^2}{3x+6}$

$= \frac{5x^2}{3(x+2)}$ $\quad x+2 \neq 0$, $-2 \quad -2$, $x \neq -2$

Domain: All real numbers, except $x \neq -2$

Same as $(-\infty, -2) \cup (-2, \infty)$

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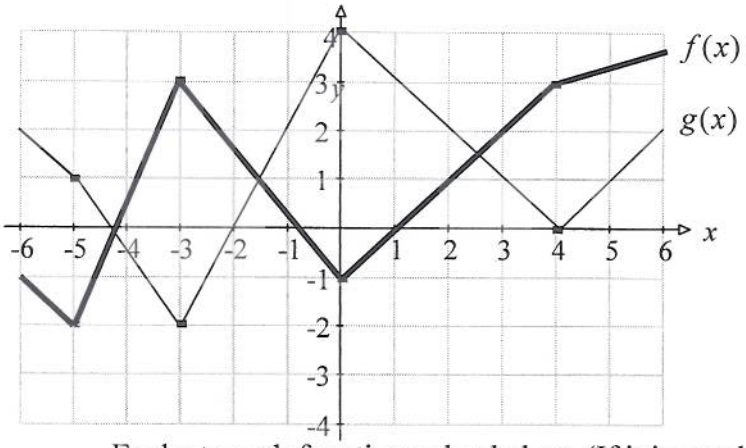
Example: Given the following graphs for $f(x)$ and $g(x)$.



Evaluate each function value below. (If it is not defined, say so.)

a) $(f+g)(0) = f(0) + g(0) = -1 + 4 = 3$

b) $(f-g)(-3) = f(-3) - g(-3) = 3 - (-2) = 3 + 2 = 5$

c) $(fg)(2) = f(2) \cdot g(2) = 1 \cdot 2 = 2$

d) $\left(\frac{f}{g}\right)(4) = \frac{f(4)}{g(4)} = \frac{3}{0}$ ← not defined

e) $\left(\frac{f}{g}\right)(-5) = \frac{f(-5)}{g(-5)} = \frac{-2}{1} = -2$