MAT 012 Review / Lecture Notes Ch 3.6 & 7.1: Introduction to Functions

START OF REVIEW (not covered during lecture)

Function:
Relationship of two variables. For every value of the first variable there is only one corresponding value of the second variable.

Representing Functions:
A Function can be represented in

- A data table
- A graph
- An equation

Functions tell us specifically how one quantity varies with respect to another quantity (or other quantities.) The quantities related by a function are called variables because they change, or vary.

Independent variable:

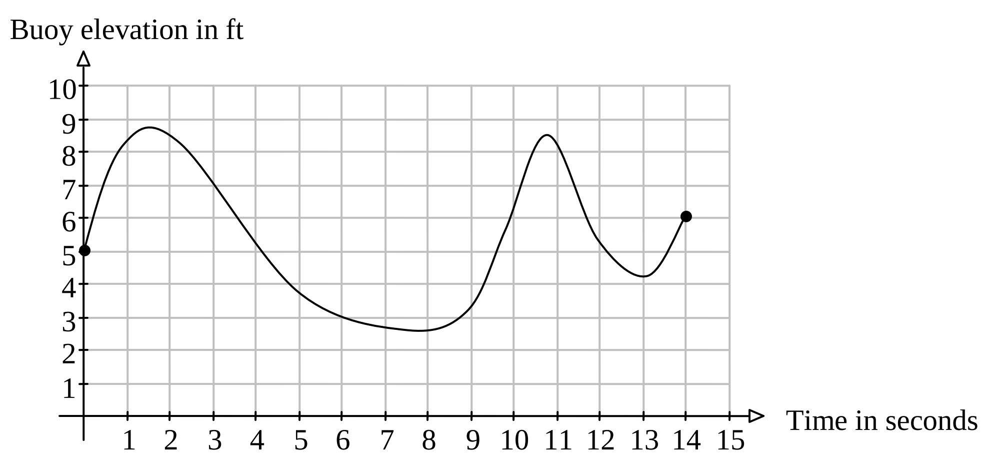
- The variable plotted on the *x*-axis.
- The first entry in an ordered pair.
- This variable can often be controlled or manipulated but is not dependent on the other variable. (*Example*: The number of hours you work in your part-time job in a certain week.)
- Note: Time (Time is usually considered an independent variable.)

Dependent variable:

- The variable plotted on the *y*-axis.
- The second entry in an ordered pair.
- This is the variable that cannot be controlled or manipulated. It is a clear response to the input. It is the output. (*Example*: The dollar amount you get paid in your part-time job in a certain week.)

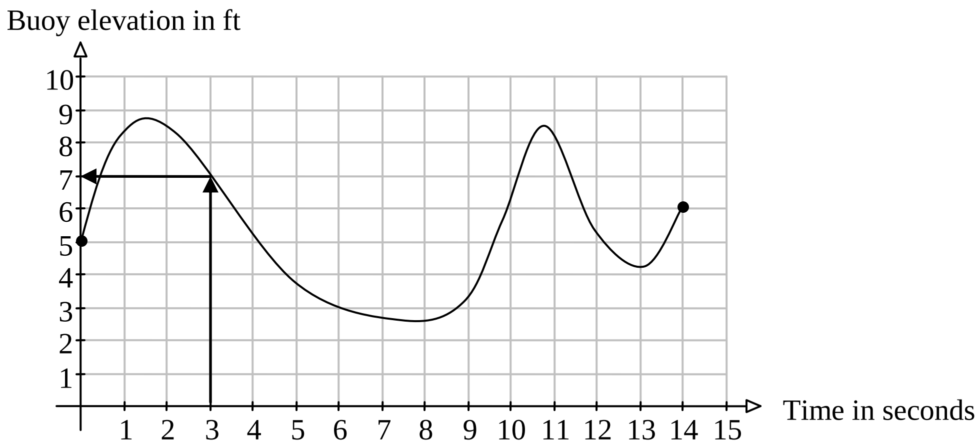
Predictions from the graph of a function

Example: A buoy is bouncing up and down on the waves. The following graph depicts its elevation over a certain period of time.



What is the buoy elevation after 3 seconds?

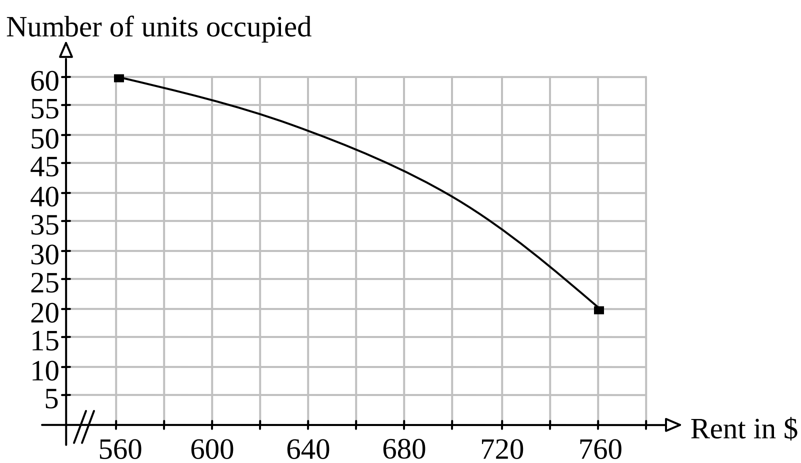
Look up from $x = 3$ *until you hit the graph, then over to the left*.



After 3 seconds, the buoy is **7 ft** high.

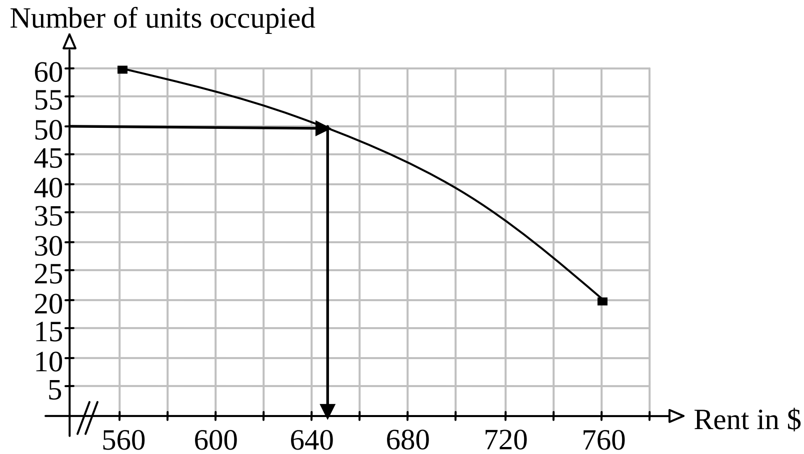
To find a *y*-value for a **given *x*-value** using a graph located in the first quadrant: Move up from the given *x*-value on the *x*-axis to the graph and then left to the *y*-axis (up and over).

Example: A real estate agent handles an apartment complex in Jacksonville, North Carolina, with 60 units. All apartments are 2 bedroom units and are rented for the same price. The higher the rent asked, the more units stay vacant. On the next page, find the graph depicting the relationship between rent price in $ and the number of units occupied. (*Yes, this rent may seem cheap to you, but many people in such a rural area are making only minimum wage*.)



Estimate for which rent 50 units will be occupied.

Look to the right from y = 50 *until you hit the graph, then down to the x-axis*.



If 50 units are occupied, the rent is about **$645** per unit.

To find an *x*-value for a **given *y*-value** using a graph located in the first quadrant: Move right from the given *y*-value on the *y*-axis to the graph and then down to the *x*-axis (over and down).

END OF REVIEW

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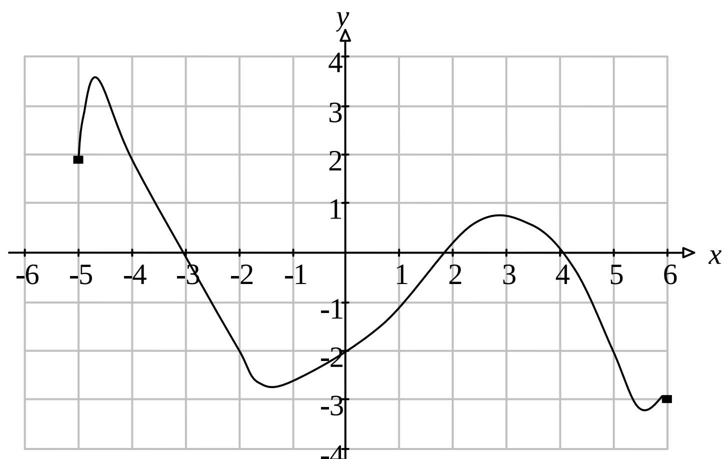
Function: Relationship of two variables. For every value of the first variable there is only one corresponding value of the second variable.

Function "name" and function notation:
A function can be named with the dependent variable name or expressed in function notation. In Algebra classes we usually use the dependent variable y or f(x) (Read: "f of x") to represent the function. f(x) is preferred by mathematicians, because it addresses the independent variable in the "name."

For example, $f(x) = 3x - 7$ is the same as writing $y = 3x - 7$. Every value x has a value of y assigned to it.

Reading function values from a graph:
Example: Give specific values for the following function graph below:



a) $f(6) =$

b) $f(0) =$

c) $f(-4) =$

d) $f(4) =$

e) Give all values of x for which $f(x) = -2$

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Evaluating Functions

Example: Let $f(x) = 3x^2 - x - 18$, evaluate $f(-2)$

Example: Let $H(t) = -t^2 + 7t - 3$

Evaluate $H(1) =$

$H(0) =$

$H(-3) =$

Example: Let $f(x) = x^2 - 5x + 9$

Find $f(a-1)$ and simplify

Example: Let $f(x) = -x^2 + 4x - 3$

Find $f(a-1)$ and simplify

Example: Let $f(x) = \dfrac{x-1}{x+3}$.

If defined, evaluate $f(-2)$, $f(-3)$, and $f(0)$. If the function is not defined for this value *x*, say so.

$f(-2) =$

$f(-3) =$

$f(0) =$

Domain and Range

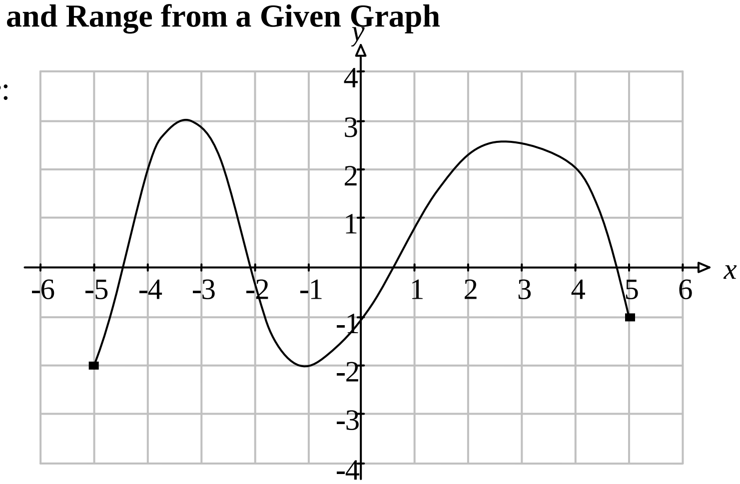
Domain: The domain of a function is the set of values for which the independent variable *x* is defined. (In some applied problems the domain may have to be restricted to a smaller interval.)

Range: The range of a function consists of the values of the dependent variable *y* that correspond to the values in the domain, i.e. which outputs are created by the *x*-values?

The domain of a function can be found by looking at the graph or the function equation. The **range** is best found looking at the graph. The intervals for domain and range must span the values that are used in the graph. Never make a domain or a range smaller than the given values.

Domain and Range from a Given Graph

Example:



Domain:

Range:

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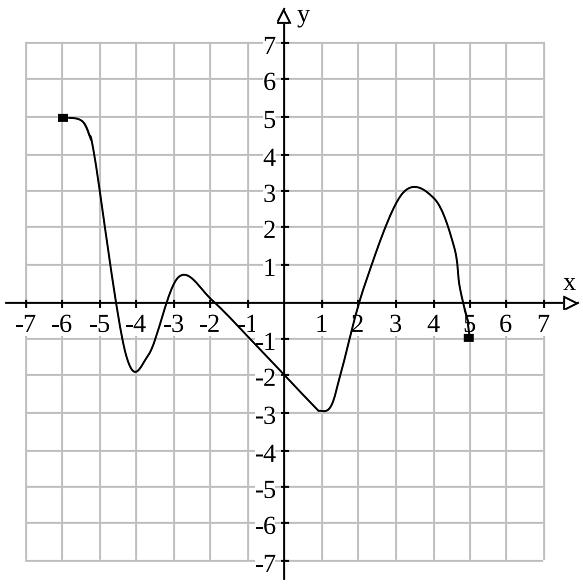
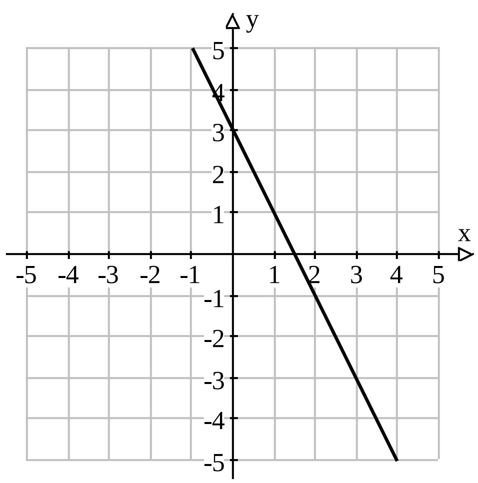
Example:



Domain:

Range:

Example: For the graph of $g(x) = -2x + 3$ shown below, give the domain and range.



Domain:

Range:

Exclude values from the domain for which the function is undefined

The **domain** of a function can also be found by examining the equation and excluding all *x* values for which f(*x*) would be undefined.

Example: Let $f(x) = \dfrac{x-1}{x+3}$. Give the domain

Domain:

Example: Let $h(x) = \dfrac{5-x}{3x-7}$. Give the domain.

Domain:

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Example: Let $f(x) = \dfrac{x-6}{x^2+5x}$. Give the domain.

Domain:

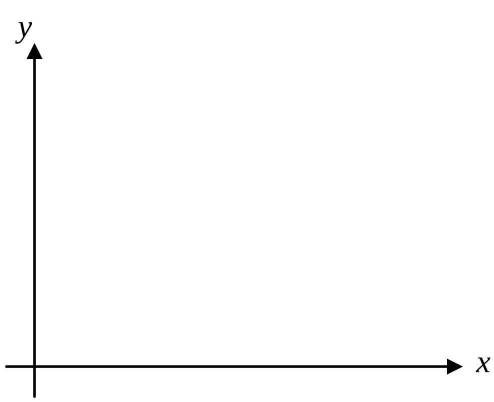
Vertical line test

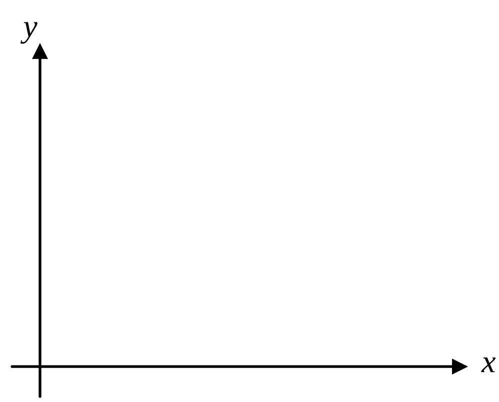
If it is possible for a vertical line to cross the graph more than once, then the graph is **not** the graph of a function.

In other words:

> If all possible vertical lines only intersect the graph at most once, then the graph represents a function.

Examples:
Determine whether this graph is the graph of a function. Use the vertical line test.





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Extra Practice: Function Algebra

1. Let $f(x) = x^2 - 5x - 4$
   a) Evaluate $f(-1)$
   b) Evaluate $f(-2)$
   c) Give and simplify $f(a+2)$

2. Let $f(x) = -x^2 + 4x + 3$
   a) Evaluate $f(1)$
   b) Evaluate $f(-2)$
   c) Give and simplify $f(a-1)$

3. Let $f(x) = 2x^2 - 7x + 4$
   a) Evaluate $f(-2)$
   b) Evaluate $f(1)$
   c) Give and simplify $f(a+1)$

4. Let $f(x) = -3x^2 - 6x - 7$
   a) Evaluate $f(-1)$
   b) Evaluate $f(-2)$
   c) Give and simplify $f(a-2)$

5. Let $f(x) = x^2 - x - 5$
   a) Evaluate $f(3)$
   b) Evaluate $f(-1)$
   c) Give and simplify $f(a+3)$

6. Let $f(x) = -x^2 + 2x + 8$
   a) Evaluate $f(4)$
   b) Evaluate $f(-1)$
   c) Give and simplify $f(a-1)$

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ANSWERS to Extra Practice: Function Algebra

1. $f(-1) = 2$; $f(-2) = 10$; $f(a+2) = a^2 - a - 10$

2. $f(1) = 6$; $f(-2) = -9$; $f(a-1) = -a^2 + 6a - 2$

3. $f(-2) = 26$; $f(1) = -1$; $f(a+1) = 2a^2 - 3a - 1$

4. $f(-1) = -4$; $f(-2) = -7$; $f(a-2) = -3a^2 + 6a - 7$

5. $f(3) = 1$; $f(-1) = -3$; $f(a+3) = a^2 + 5a + 1$

6. $f(4) = 0$; $f(-1) = 5$; $f(a-1) = -a^2 + 4a + 5$