MAT 012 Lec. Notes: ch 9.2 & 9.3, Absolute Value Equations and Inequalities

The **absolute value** produces **positive** outputs:

We know $|-6| =$

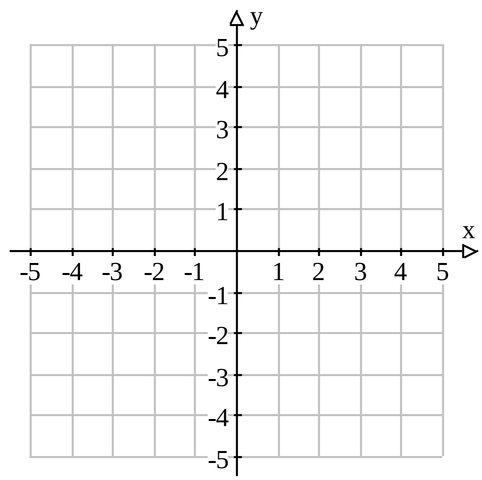
$\left|\frac{2}{3}\right| =$

$|0| =$

The **definition** of the **absolute value**: $|x| = \begin{cases} x, & if \;\; x \geq 0 \\ -x, & if \;\; x < 0 \end{cases}$

The absolute value function:

x	$f(x) = \vert x \vert$
-5	
-4	
-3	
-2	
-1	
0	
1	
2	
3	
4	
5	



Domain: **Range**:

Absolute Value Equations:

Give all solutions for which $|x| = 2$: Equation(s):

Give all solutions for which $|x| = 0$: Equation(s):

Give all solutions for which $|x| = \frac{1}{5}$: Equation(s):

Give all solutions for which $|x| = -3$: Equation(s):

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The **absolute value principle** for equations:

- First, isolate the absolute value expression
- Then, judge the case.

p is positive

Case 1: $\left| \text{An expression with } x \right| = p$

means The expression with $x = -p$ or The expression with $x = p$

Case 2: $\left| \text{An expression with } x \right| = 0$

means The expression with $x = 0$

Case 3: $\left| \text{An expression with } x \right| = -p$

Means $\varnothing$

p is positive, so $-p$ is negative

Example: Solve $|2x-3|=5$

Example: Solve $|3x+10|=-2$

Example: Solve $\left|\frac{2x-1}{3}\right|=5$

Remember: If the absolute value expression is not isolated on one side, first isolate it and then judge the case

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Example: Solve $5-2|3x-4|=-5$ ← Note: Do not claim "no solutions" since this is only true for isolated absolute values…

Example: Solve $|5x-7|-3=10$

Note: We are skipping the problems of the type $|\text{An expression with } x| = |\text{An expression with } x|$

Absolute Value Inequalities:

Example: $|x|\le 3$

List of selected numbers that work: e.g.

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Example: $|x| > 5$

List of selected numbers that work: e.g.

In general: Again, isolate the absolute value expression, then judge the type.

"Less than" type: $|\text{An expression with } x| < p$ ← p is positive

Solutions set-up: $-p <$ The expression with x $< p$

"Greater than" type: $|\text{An expression with } x| > p$ ← p is positive

Solutions set-up: The expression with $x < -p$, **or** The expression with $x > p$

<u>Note</u>: The "less than" type is also applied for $|\text{An expression with } x| \leq p$. Then the solution set-up uses "less than or equal to" signs.

<u>Note</u>: The "greater than" type is also applied for $|\text{An expression with } x| \geq p$. Then the solution set-up uses "less than or equal to" and "greater than or equal to" respectively.

<u>Note</u>: In addition to the types mentioned above, there are some special cases, which are discussed at the end of this handout.

Example: Solve, graph on the number line and give in interval notation:
$|x+4| \leq 1$

Interval notation:

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Example: Solve, graph on the number line and give in interval notation:

$|3x-4|>8$

Interval notation:

Example: Solve, graph on the number line and give in interval notation:

$30-4|x+2|>12$

Interval notation:

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Example: Solve, graph on the number line and give in interval notation:

$|4x+10|-7 \geq 11$

Interval notation:

Special cases of Absolute Value Inequalities

Example: $|x| > -2$

Example: $|x| \leq -6$

In general:

$|\text{An expression with } x| > -p$ ← p is positive, so $-p$ is negative

Is always correct, since a positive number is always greater than a negative number.

Solutions: **All real numbers**.

$|\text{An expression with } x| < -p$ ← p is positive, so $-p$ is negative

Is never correct, since a positive number can never be smaller than a negative one.

Solutions: **No solutions**.

Example: Solve $|7-2x| < -4$

Example: Solve $|x-4|+5 \geq 2$