MAT 137 **Logarithms**

Since the exponential function is one-to-one (it passes the horizontal line test), it has an inverse function. This inverse is the logarithmic function.

Solving an exponential function for the exponent produces the logarithm.

$\log_b c = x$	is equivalent to	$b^x = c$
When working with logarithms, rewrite them in exponential form, then evaluate.		

Examples:

1) Evaluate: $\log_2 16 = x$

$2^x = 16$

$x = 4$ $\qquad \log_2 16 = 4$

2) Evaluate: $\log_2 \frac{1}{4} = x$

$2^x = \frac{1}{4}$

$x = -2$ $\qquad \log_2 \frac{1}{4} = -2$

3) Evaluate: $\log_{10} 10{,}000 = x$

$10^x = 10{,}000$

$x = 4$ $\qquad \log_{10} 10{,}000 = 4$

4) Evaluate: $\log_2(-4) = x$

$2^x = -4$ $\qquad \log_2(-4)$ is undefined

Does not exist

5) Find x: $\log_2 x = -3$

$2^{-3} = x$

$\frac{1}{2^3} = x$ $\qquad x = \frac{1}{8}$

6) Find x: $\log_x 125 = 3$

$x^3 = 125$

$x = 5$ $\qquad x = 5$

7) Find x: $\log_{10} x = 5$

$10^5 = x$

$100{,}000 = x$ $\qquad x = 100{,}000$

8) Find x: $\log_7 x = 0$

$7^0 = x$

$1 = x$ $\quad x = 1$

9) Evaluate: $\log_{10} 0.001 = x$

$10^x = 0.001$

$x = -3$ $\quad \log_{10} 0.001 = -3$

10) Evaluate: $\log_5 5 = x$

$5^x = 5$

$x = 1$ $\quad \log_5 5 = 1$

11) Find x: $\log_4 x = 1$

$4^1 = x$

$4 = x$ $\quad x = 4$

12) Find x: $\log_x 32 = 5$

$x^5 = 32$

$x = 2$ $\quad x = 2$

13) Evaluate $\log_{10} \frac{1}{1000} = x$

$10^x = \frac{1}{1000}$

$x = -3$ $\quad \log_{10} \frac{1}{1000} = -3$

14) Find x: $\log_9 x = \frac{1}{2}$

$9^{\frac{1}{2}} = x$

$\sqrt{9} = x$ $\quad x = 3$

15) Find x: $\log_{10} x = -2$

$10^{-2} = x$

$\frac{1}{100} = x$ $\quad x = \frac{1}{100}$ same as $x = 0.01$

16) Evaluate: $\log_6 1 = x$

$6^x = 1$

$x = 0$ $\quad \log_6 1 = 0$

17) Find x: $\log_x \frac{1}{49} = -2$

$x^{-2} = \frac{1}{49}$

$x = 7$ $x = 7$

18) Evaluate: $\log_{\frac{1}{4}} 64 = x$

$\left(\frac{1}{4}\right)^x = 64$

$x = -3$ $\log_{\frac{1}{4}} 64 = -3$

19) Find x: $\log_{\frac{1}{3}} x = -4$

$\left(\frac{1}{3}\right)^{-4} = x$

$3^4 = x$ $x = 81$

20) Evaluate: $\log_8 \sqrt{8} = x$

$8^x = \sqrt{8}$

$8^x = 8^{\frac{1}{2}}$

$x = \frac{1}{2}$ $\log_8 \sqrt{8} = \frac{1}{2}$

21) Find x: $\log_x \frac{81}{16} = -4$

$x^{-4} = \frac{81}{16}$

$x^4 = \frac{16}{81}$ $x = \frac{2}{3}$

Often base 10 is implied. By convention: $\log_{10} x$ is written as $\log x$.

If the so called Euler number $e = $ **2.7182818284…** (pronounce "Oiler number") is used as a base in a logarithm, then we call this logarithm base e the ***natural logarithm***. The convention for this logarithm is that $\log_e x$ is written as $\ln x$.

Logarithms on the calculator

Base **10** $\boxed{\text{LOG}}$ key: to evaluate $\log_{10} 10{,}000$, type log(10,000) =

to evaluate $\log_{10} 0.00001$, type log(0.00001) =

Base e $\boxed{\text{LN}}$ key to evaluate $\ln 7$ type ln(7) for a decimal approximation.

If you need it exact, leave $\ln 7$

Evaluating logarithms other than base 10 or base e

A useful relationship called the base change formula:

$\boxed{\log_b c = \dfrac{\log_{10} c}{\log_{10} b} \text{ which means using the "LOG" button type: } \dfrac{\log(c)}{\log(b)}}$ or alternatively $\dfrac{\ln(c)}{\ln(b)}$ with the "LN" button

Alternative, use $\boxed{\text{MATH}}$ option "log BASE"
or use $\boxed{\text{MATH}}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{MATH}}$ on the graphing calculator

Example:

22) Evaluate: $\log_2 128 = 7$

e.g. type $\dfrac{\log(128)}{\log(2)}$ on the calculator or solve $2^x = 128$, $x = 7$

23) Evaluate: $\log_2(-2)$ is undefined

e.g. type $\dfrac{\log(-2)}{\log(2)}$ on the calculator or judge $2^x = -2$ does not exist

24) Evaluate: *Give an exact answer*: $\log_{\frac{1}{2}} \sqrt[3]{2} = -\frac{1}{3}$

e.g. type $\dfrac{\log(\sqrt[3]{2})}{\log(\frac{1}{2})}$ on the calculator or solve $\left(\frac{1}{2}\right)^x = \sqrt[3]{2}$, $\left(\frac{1}{2}\right)^x = 2^{\frac{1}{3}}$, $x = -\frac{1}{3}$

Graphing logarithms other than base 10 or base e **with the graphing calculator**

To graph logarithms that are not base 10 or base e, use the same base change formula as above:

25) Graph on a graphing calculator: $f(x) = -\log_3 x$ for "y=" type $\dfrac{-\log(x)}{\log(3)}$

26) Graph on a graphing calculator: $g(x) = \log_{\frac{1}{2}}(x+4)$ for "y=" type $\dfrac{\log(x+4)}{\log(\frac{1}{2})}$