

Since the exponential function is one-to-one (it passes the horizontal line test), it has an inverse function. This inverse is the logarithmic function.

Solving an exponential function for the exponent produces the logarithm.

$$\log_b c = x \quad \text{is equivalent to} \quad b^x = c$$

When working with logarithms, rewrite them in exponential form, then evaluate.

Examples:

1) Evaluate: $\log_2 16$

2) Evaluate: $\log_2 \frac{1}{4}$

3) Evaluate: $\log_{10} 10,000$

4) Evaluate: $\log_2(-4)$

5) Find x : $\log_2 x = -3$

6) Find x : $\log_x 125 = 3$

7) Find x : $\log_{10} x = 5$

8) Find x : $\log_7 x = 0$

9) Evaluate: $\log_{10} 0.001$

10) Evaluate: $\log_5 5$

11) Find x : $\log_4 x = 1$

12) Find x : $\log_x 32 = 5$

13) Evaluate $\log_{10} \frac{1}{1000}$

14) Find x : $\log_9 x = \frac{1}{2}$

15) Find x : $\log_{10} x = -2$

16) Evaluate: $\log_6 1$

17) Find x : $\log_x \frac{1}{49} = -2$

18) Evaluate: $\log_{\frac{1}{4}} 64$

19) Find x : $\log_{\frac{1}{3}} x = -4$

20) Evaluate: $\log_8 \sqrt{8}$

21) Find x : $\log_x \frac{81}{16} = -4$

Often base 10 is implied. By convention: $\log_{10} x$ is written as $\log x$.

If the so called Euler number $e = 2.7182818284\dots$ (pronounce “Oiler number”) is used as a base in a logarithm, then we call this logarithm base e the *natural logarithm*. The convention for this logarithm is that $\log_e x$ is written as $\ln x$.

Logarithms on the calculator

Base 10 LOG key: to evaluate $\log_{10} 10,000$, type $\log(10,000) =$

to evaluate $\log_{10} 0.00001$, type $\log(0.00001) =$

Base e LN key to evaluate $\ln 7$ type $\ln(7)$ for a decimal approximation.

If you need it exact, leave $\ln 7$

Evaluating logarithms other than base 10 or base e

A useful relationship called the base change formula:

$$\log_b c = \frac{\log_{10} c}{\log_{10} b} \text{ or alternatively } \frac{\ln c}{\ln b}$$

Example:

22) Evaluate: $\log_2 128$

23) Evaluate: $\log_2(-2)$

24) Evaluate: *Give an exact answer:* $\log_{\frac{1}{2}} \sqrt[3]{2}$

Graphing logarithms other than base 10 or base e with the graphing calculator

To graph logarithms that are not base 10 or base e , use the same base change formula as above:

25) Graph on a graphing calculator: $f(x) = -\log_3 x$

26) Graph on a graphing calculator: $g(x) = \log_{\frac{1}{2}}(x+4)$