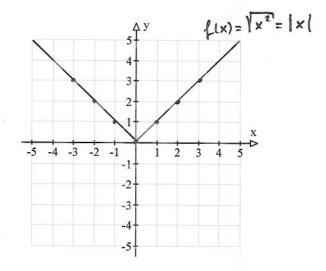
Let's look at:



And at

Think about the following functions and verify them with your graphing calculator:

$$f(x) = \sqrt[4]{x^4} = |x|$$

$$f(x) = \sqrt[5]{x^5} = \mathbf{x}$$

$$f(x) = \sqrt[6]{x^6} = |x|$$

## MAT 012

## Lecture Notes, ch 10, Suppl. B: Radical Expressions

So for all even  $n \in \mathbb{N}$ ,  $n \ge 2$ :

$$f(x) = \sqrt[n]{x^n} = |x|$$

And for all odd  $n \in N$ ,  $n \ge 3$ :

$$f(x) = \sqrt[n]{x^n} = x$$

The same holds true for "expressions with x" in the radicand:

$$f(x) = \sqrt{(\text{expression with } x)^2} = |\text{expression with } x|$$

and in general:

For all even  $n \in \mathbb{N}$ ,  $n \ge 2$ :

$$f(x) = \sqrt[n]{(\text{expression with } x)^n} = |\text{expression with } x|$$

And for all odd  $n \in N$ ,  $n \ge 3$ :

$$f(x) = \sqrt[n]{(\text{expression with } x)^n} = \text{expression with } x$$

Example: Simplify 
$$d(x) = \sqrt{(x-4)^2} = |x-4|$$

Example: Simplify 
$$k(x) = \sqrt[3]{-8x^3} = \sqrt[3]{(-2x)^3} = -2x$$

Example: Simplify 
$$h(x) = \sqrt[6]{729x^6} = \sqrt{(3x)^2} = |3x|$$

Example: Simplify 
$$g(x) = \sqrt{16x^6} = \sqrt{4^2(x^3)^2} = \sqrt{(4x^3)^2} = |4x^3|$$

Sometimes we can simplify further by asking if it is actually necessary to keep the absolute value in the answer. Let's look at the following examples:

Example:

Simplify 
$$f(x) = \sqrt{9x^4} = \sqrt{3^2(x^2)^2} = \sqrt{(3 \times 9^2)^2} = |3x^2| = 3x^2$$

Example:

Simplify 
$$f(x) = \sqrt[4]{x^{24}} = \sqrt[4]{(x^{4})^{4}} = |x^{4}| = |x^{4}|$$

X' is always positive Often, we will just consider positive radicands, which eliminates the need for an absolute value. So read the instructions to each problem you are about to work carefully. Here is what you will see if you do NOT have to consider absolute values when simplifying even radicals:

Example: Simplify the following expression. Assume that the variable is positive:

$$\sqrt{36a^{10}} = 6a^{5}$$

Example: Simplify the following expression. Assume that the expression with x is positive: (Which means that you may ignore the issue of the absolute value in this problem.)

$$\sqrt{x^2-10x+25} = \sqrt{(x-5)(x-5)} = \sqrt{(x-5)^2} = x-5$$