Solving radical equations that involve square roots

Steps:

- If the radical expression was a function, what would it's domain be?

- Isolate the radical. (Make sure the isolated radical is positive.)

- Square both sides.

- Don't forget to FOIL where appropriate

- Solve (often includes factoring)

- Always check for extraneous solutions (there are solutions that are introduced by squaring both sides that do NOT check).
  Note: Check in the original version, never after you squared both sides.

Why is the check necessary as part of the method?

square both sides $\left\{\begin{array}{l} X = 4 \quad \text{has the solution} \quad x = 4 \\ \text{but } x^2 = 16 \text{ has the two solutions } x = -4 \text{ or } x = 4 \end{array}\right\}$ not equivalent

Example: a) Find the domain of $f(x) = \sqrt{x-1}$ Domain:

$$X - 1 \geq 0$$
$$\underline{+1 \quad +1}$$
$$X \geq 1$$

$X \geq 1$

b) Solve $x = \sqrt{x-1} + 3$

$$\underline{-3 \qquad -3}$$

$$(x-3)^2 = (\sqrt{x-1})^2$$

$$(x-3)(x-3) = x - 1$$

$$x^2 - 6x + 9 = x - 1$$
$$\underline{-x + 1 \qquad -x + 1}$$

$$x^2 - 7x + 10 = 0$$

$$(x-5)(x-2) = 0$$

$$\boxed{X = 5} \text{ or } x \neq 2$$

Check $x = 5$ in $\quad x - 3 \overset{?}{=} \sqrt{x-1}$

$$5 - 3 \overset{?}{=} \sqrt{5-1}$$
$$2 \overset{?}{=} \sqrt{4}$$
$$2 = 2 \checkmark$$

Check $x = 2$ in $\quad x - 3 = \sqrt{x-1}$

$$2 - 3 \overset{?}{=} \sqrt{2-1}$$
$$-1 \overset{?}{=} \sqrt{1}$$
$$-1 \neq 1$$

Answer: $x = 5$

Example: a) Find the domain of $f(x) = \sqrt{4-x}$

$$4 - x \geq 0$$
$$\underline{-4 \qquad\qquad -4}$$
$$\frac{-x}{-1} \geq \frac{-4}{-1}$$
$$x \leq -4$$

b) Solve $2 + \sqrt{4-x} = x$

$$\underline{-2 \qquad\qquad -2}$$

$$\sqrt{4-x} = x - 2$$

$$\left(\sqrt{4-x}\right)^2 = (x-2)^2$$

$$4 - x = (x-2)(x-2)$$

$$4 - x = x^2 - 4x + 4$$
$$\underline{-4 + x \qquad\qquad + x \quad -4}$$

$$0 = x^2 - 3x$$

$$0 = x(x-3)$$

$$x \neq 0 \quad \text{or} \quad \boxed{x = 3}$$

Check $x = 0$ in $\sqrt{4-x} = x - 2$

$$\sqrt{4-0} \stackrel{?}{=} 0 - 2$$
$$\sqrt{4} \stackrel{?}{=} -2$$
$$2 \neq -2$$

extraneous solution, so $x \neq 0$

Check $x = 3$ in $\sqrt{4-x} = x-2$

$$\sqrt{4-3} \stackrel{?}{=} 3 - 2$$
$$\sqrt{1} \stackrel{?}{=} 1$$
$$1 = 1 \checkmark$$

Answer: $\boxed{x = 3}$

Example: a) Find the domain of $f(x) = \sqrt{3x+7}$

$$3x + 7 \geq 0$$
$$\underline{-7 \quad -7}$$
$$\frac{3x}{3} \geq \frac{-7}{3}$$
$$x \geq -\frac{7}{3}$$

Domain : $x \geq -\frac{7}{3}$

b) Solve $\sqrt{3x+7} - 3 = x$

$$\underline{\phantom{\sqrt{3x+7}}+3 \quad +3}$$
$$\left(\sqrt{3x+7}\right)^2 = (x+3)^2$$
$$3x + 7 = x^2 + 6x + 9$$
$$\underline{-3x - 7 \qquad -3x - 7}$$
$$0 = x^2 + 3x + 2$$
$$0 = (x+2)(x+1)$$
$$\boxed{x = -2 \quad \text{or} \quad x = -1}$$

Check $x = -2$ in $\sqrt{3x+7} = x+3$

$$\sqrt{3(-2)+7} \overset{?}{=} -2+3$$
$$\sqrt{-6+7} \overset{?}{=} 1$$
$$\sqrt{1} \overset{?}{=} 1$$
$$1 = 1 \checkmark$$

Check $x = -1$ in $\sqrt{3x+7} = x+3$

$$\sqrt{-3+7} \overset{?}{=} -1+3$$
$$\sqrt{4} \overset{?}{=} 2$$
$$2 = 2 \checkmark$$

Answers: $\boxed{x = -2 \quad \text{or} \quad x = -1}$