MAT 012 **Lecture Notes 11**

Example: $\boxed{f(x) = x^2 + 2x - 3}$

a) vertex (Give the point):

$$h = \frac{-b}{2a} = \frac{-2}{2} = -1$$

$$k = f(-1) = (-1)^2 + 2(-1) - 3 = 1 - 2 - 3 = -4$$

vertex : $(-1, -4)$

b) equation of the axis of symmetry: $x = -1$

c) vertex-form:

$$f(x) = (x - (-1))^2 - 4 = (x+1)^2 - 4$$

d) y-intercept (Give the point):

$$f(0) = 0^2 + 2 \cdot 0 - 3 = -3 \qquad (0, -3)$$

e) x-intercepts (Give the points if they exist):

$$x^2 + 2x - 3 = 0$$

$$(x-1)(x+3) = 0$$

$$x = 1 \qquad x = -3$$

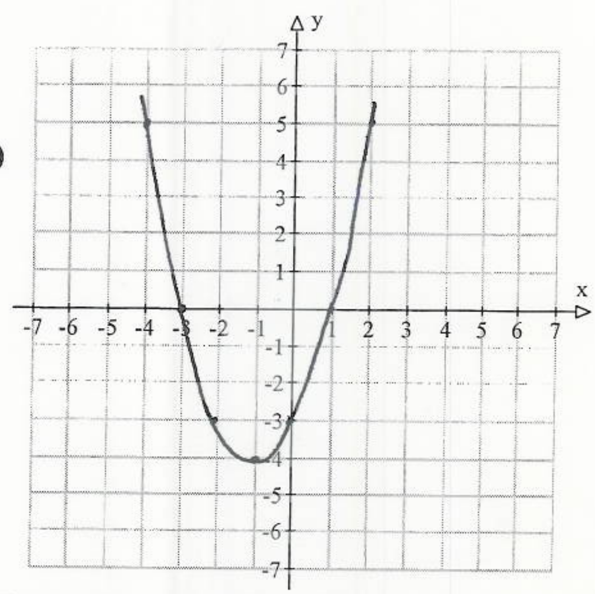
$$(-1, 0) \qquad (-3, 0)$$

f)

$$f(-2) = (-2)^2 + 2(-2) - 3$$
$$= 4 - 4 - 3$$
$$= -3 \qquad (-2, -3)$$

$$f(2) = 4 + 4 - 3 = 5 \qquad (2, 5)$$

$$f(-4) = 16 - 8 - 3 = 5 \qquad (-4, 5)$$



g) This parabola opens <u>upward</u>. This vertex is a <u>minimum</u>.
To obtain this function, x^2 is <u>not stretched</u> Here the "stretch-factor" is 1.
The parabola is shifted <u>1</u> unit~~s~~ to the <u>left</u> (horizontal shift)
and <u>4</u> units <u>down</u> (vertical shift).

Example: $\boxed{f(x)=2x^2-8x+6}$ $a=2;\ b=-8;\ c=6$

a) vertex:

$$h=\frac{-b}{2a}=\frac{8}{4}=2$$

vertex: $(2,-2)$

$$k=f(2)=2\cdot 2^2-8\cdot 2+6=2\cdot 4-16+6=8-16+6=-2$$

b) equation of the axis of symmetry: $x=2$

c) vertex-form:

$$f(x)=2(x-2)^2-2$$

d) y-intercept (Give the point):

$f(0)=6$ $(0,6)$

e) x-intercepts (Give the points if they exist):

$2x^2-8x+6=0$ divide by 2

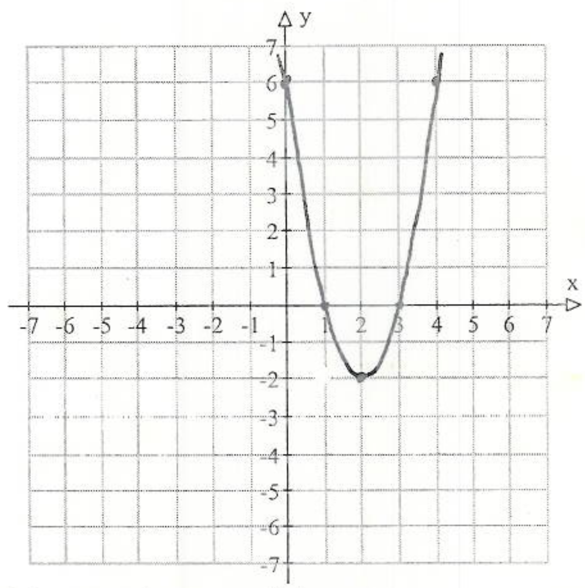
$x^2-4x+3=0$

$(x-1)(x-3)=0$

$x=1$ or $x=3$ points: $(1,0)$ $(3,0)$

f)

by symmetry: $(4,6)$



g) This parabola opens upward. This vertex is a minimum.

To obtain this function, x^2 is stretched by the "stretch-factor" 2.

The parabola is shifted 2 units to the right (horizontal shift)

and 2 units down (vertical shift).

Example: $\boxed{f(x) = -2x^2 + 12x - 14}$

a) vertex:

$$h = \frac{-b}{2a} = \frac{-12}{-4} = 3$$

$$k = f(3) = -2 \cdot 9 + 12 \cdot 3 - 14 = -18 + 36 - 14 = 4$$

vertex $(3, 4)$

b) equation of the axis of symmetry: $x = 3$

c) vertex-form:

$$f(x) = -2(x-3)^2 + 4$$

d) y-intercept:

$$f(0) = -2 \cdot 0^2 + 12 \cdot 0 - 14 = -14 \qquad (0, -14)$$

e) x-intercepts:

$$-2x^2 + 12x - 14 = 0 \quad \text{divide by } -2$$

$$x^2 - 6x + 7 = 0 \quad \text{does NOT factor} \quad a = 1, b = -6, c = 7$$

$$x = \frac{6 \pm \sqrt{36 - 4 \cdot 1 \cdot 7}}{2} = \frac{6 \pm \sqrt{36 - 28}}{2} = \frac{6 \pm \sqrt{8}}{2}$$

$$x_1 = \frac{6 + \sqrt{8}}{2} \approx 4.41$$

$$x_2 = \frac{6 - \sqrt{8}}{2} \approx 1.59$$

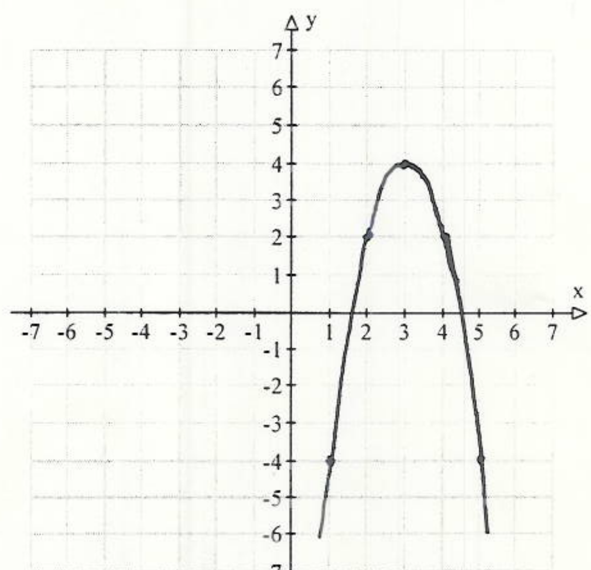
f)

$$f(2) = -2 \cdot 4 + 24 - 14$$
$$= -8 + 24 - 14 = 2$$

$$f(4) = -2 \cdot 16 + 48 - 14$$
$$= -32 + 48 - 14 = 2$$

$$f(5) = -2 \cdot 25 + 60 - 14$$
$$= -50 + 60 - 14 = -4$$

$$f(1) = -2 + 12 - 14 = -4$$



g) This parabola opens downward. This vertex is a maximum.

To obtain this function, x^2 is stretched by the "stretch-factor" 2.

The parabola is shifted 3 units to the right (horizontal shift).

and 4 units up (vertical shift).

 SOLUTIONS

Example: $\boxed{f(x) = \frac{1}{2}x^2 + 2x + 3}$ $a = \frac{1}{2}$; $b = 2$; $c = 3$

a) vertex:

$$h = \frac{-b}{2a} = \frac{-2}{2 \cdot \frac{1}{2}} = \frac{-2}{1} = -2$$

vertex: $(-2, 1)$

$$k = f(-2) = \tfrac{1}{2}(-2)^2 + 2(-2) + 3 = \tfrac{1}{2} \cdot 4 - 4 + 3 = 2 - 4 + 3 = 1$$

b) equation of the axis of symmetry: $x = -2$

c) vertex-form:

$$f(x) = \tfrac{1}{2}(x+2)^2 + 1$$

d) y-intercept (Give the point):

$f(0) = 3$ $(0, 3)$

e) x-intercepts (Give the points if they exist):

$\frac{1}{2}x^2 + 2x + 3 = 0$ multiply by $\frac{1}{2}$

$x^2 + 4x + 6 = 0$ Does <u>not</u> factor

$$x = \frac{-4 \pm \sqrt{16 - 4 \cdot 1 \cdot 6}}{2} = \frac{-4 \pm \sqrt{16-24}}{2} = \frac{-4 \pm \sqrt{-8}}{2}$$

No real solutions
$\Rightarrow$ <u>no</u> x-intercepts

f)

$(-4, 3)$ by symmetry

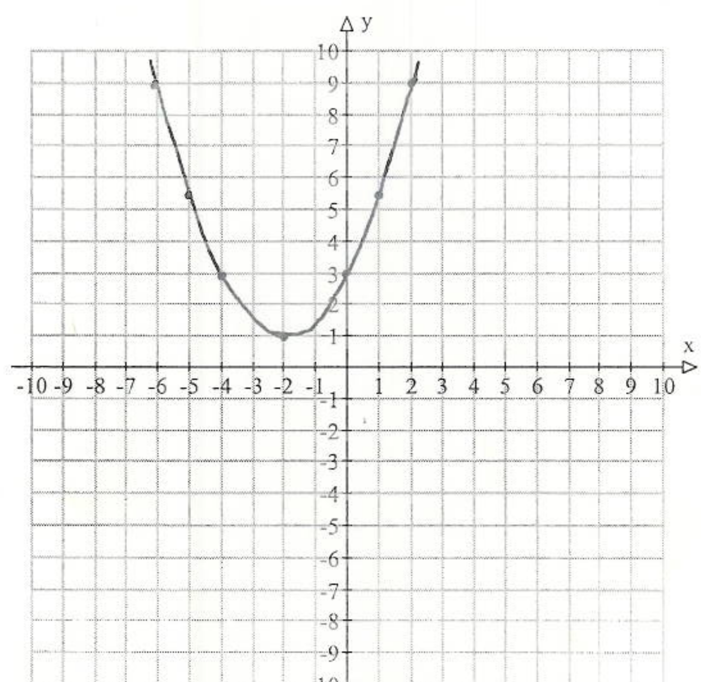
$f(1) = \frac{1}{2} \cdot 1^2 + 2 \cdot 1 + 3 = \frac{1}{2} + 2 + 3 = 5\frac{1}{2}$

by symmetry: $(-5, 5\frac{1}{2})$

$f(2) = \frac{1}{2} \cdot 2^2 + 2 \cdot 2 + 3 = \frac{1}{2} \cdot 4 + 4 + 3$

$= 2 + 4 + 3 = 9$

by symmetry: $(-6, 9)$



g) This parabola opens <u>upward</u>.

This vertex is a <u>minimum</u>. To obtain this function, x^2 is <u>Widened / squeezed</u> by the "stretch-factor" <u>$\frac{1}{2}$</u>. The parabola is shifted <u>2</u> units to the <u>left</u> (horizontal shift) and <u>1</u> units <u>up</u> (vertical shift).

Back to the first type of exercises: If the vertex form of a function is given, we know a lot about the resulting parabola just by looking at the function.

Example: $\boxed{f(x) = -\frac{1}{4}(x+6)^2 + 8}$

a) This parabola opens _downward_. This vertex is a _maximum_
To obtain this function, x^2 is _squeezed/widened_ by the "stretch-factor" _$\frac{1}{4}$_.
The parabola is shifted _6_ units to the _left_ (horizontal shift)
and _8_ units _up_ (vertical shift).
The vertex of the parabola is the point _$(-6, 8)$_.

b) Convert the given vertex form to the standard form $f(x) = ax^2 + bx + c$.

$$f(x) = -\frac{1}{4}\left[(x+6)(x+6)\right] + 8$$
$$= -\frac{1}{4}(x^2 + 6x + 6x + 36) + 8$$
$$= -\frac{1}{4}(x^2 + 12x + 36) + 8$$
$$= -\frac{1}{4}x^2 - 3x - 9 + 8$$
$$= -\frac{1}{4}x^2 - 3x - 1$$

Example: $\boxed{f(x) = 4(x-2)^2 - 7}$

a) This parabola opens _upword_. This vertex is a _minimum_
To obtain this function, x^2 is _stretched/narrowed_ by the "stretch-factor" _4_.
The parabola is shifted _2_ units to the _right_ (horizontal shift).
and _7_ units _down_ (vertical shift)
The vertex of the parabola is the point _$(2, -7)$_.

b) Convert the given vertex form to the standard form $f(x) = ax^2 + bx + c$.

$$f(x) = 4\left[(x-2)(x-2)\right] - 7$$
$$= 4(x^2 - 2x - 2x + 4) - 7$$
$$= 4(x^2 - 4x + 4) - 7$$
$$= 4x^2 - 16x + 16 - 7$$
$$= 4x^2 - 16x + 9$$