MAT 012 Ch 11, Quadratic Functions (Theory)

For each given parabola in standard form $f(x) = ax^2 + bx + c$, we will

a) Find the vertex. (Give the point).

- To find the *x*- value of the vertex, *h*, use the formula: $h = \frac{-b}{2a}$
- $f(h) = k$: To find the *y*-value of the vertex, *k*, plug *h* back into the given function f and evaluate.

List the vertex (h, k)

b) Give the axis of symmetry.

Use the equation $x = h$ with the appropriate number *h* for the vertical axis of symmetry.

c) Use the vertex to bring the equation in the vertex-form $f(x) = a(x-h)^2 + k$ **.**
Use *a* from the standard form.

Substitute *a*, as well as *h* and *k* from above into the formula $f(x) = a(x-h)^2 + k$

<u>Note</u>: Simplify the signs in the expression of $(x-h)^2$. For negative numbers *h* this becomes an addition.

d) Find the *y*-intercept. (Give the point).

Plug $x = 0$ into the standard form. *c* from the standard form is the *y*-value.

Give the entire point, not just the *y*-value.

e) Find the *x*-intercepts for the function. If it does not have any, say so. (Otherwise, give the points)

Since $y = 0$, set the quadratic function equal to zero. (Do not forget to show the "= 0")

Solve for *x* by factoring or use the quadratic formula. If the equation factors, you can use either method. If it does not factor, you must use the quadratic formula to solve and give approximate decimal answers (if the function has *x*-intercepts). Keep in mind that there may be quadratic functions with no *x*-intercepts.
Give the entire points, not just the *x*-values.

f) Sketch the graph

Use the vertex, key points, *x*-intercepts and the *y*-intercept (*y*-intercept only if in "graphing window".) Calculate additional key points by selecting integer *x*-values to find points on the graph.

Consider to use symmetry, so you only have to calculate half the number of key points.

g) Describe how the new graph is obtained from the parabola $g(x) = x^2$ by answering the following questions:

- **Does the parabola open upward or downward? (Does the vertex represent a maximum or a minimum?)**
  Analyze *a* from the vertex form or from the standard form:
  If *a* is positive, the parabola opens upward and the vertex is a minimum.
  If *a* is negative, the parabola opens downward and the vertex is a maximum.

- **Is the parabola stretched (narrower, slimmer) or squeezed (wider) than the graph of** $g(x) = x^2$ **? Which "stretch-factor", a, was used [if applicable]?**
  Again, analyze *a* from the vertex form or from the standard form:
  If $|a| > 1$, the parabola is stretched in *y*-direction (slimmer, narrower) with the stretch factor of $|a|$.
  If $-1 < a < 1$, the parabola is squeezed in *y*-direction (wider) with the stretch factor of $|a|$.
  If $a = 1$ the parabola is neither stretched, nor squeezed.

- **Is the parabola shifted in *x* direction (horizontal shift). If so, how many units and in which direction (left or right)?**
  Analyze the *x*-value of the vertex, *h*, from your vertex calculation or in the vertex formula.
  Note: There is a minus build into the vertex formula, so make sure you report the shift in the correct direction.

- **Is the parabola shifted in *y* direction (vertical shift). If so, how many units and in which direction (up or down)?**
  Analyze the *y*-value of the vertex, *k*, from your vertex calculation or in the vertex formula.

- **[If asked] give the vertex of the parabola.**
  Analyze *h* and *k* from the vertex form.
  The vertex of the parabola is the point (h, k).
  Note: There is a minus build into the vertex formula, so make sure your *h* has the correct sign.