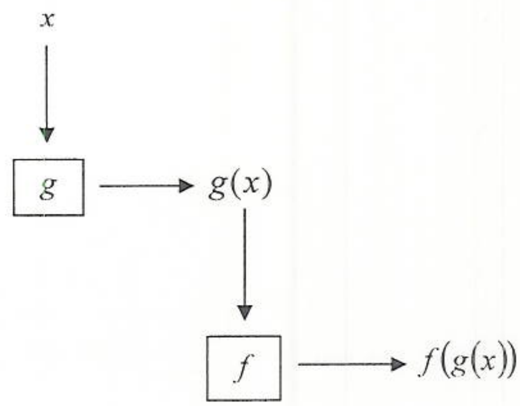
MAT 012 Lecture Notes : Composite Functions

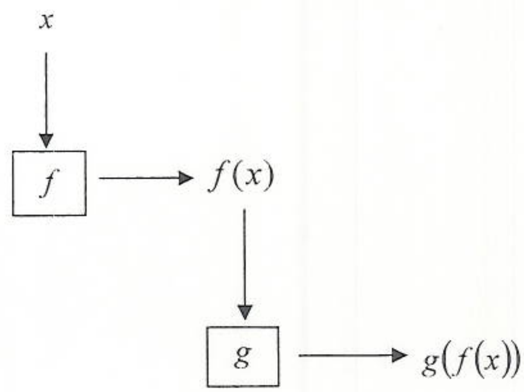
Composite Functions:

Let $f(x)$ and $g(x)$ be given functions, then

$\boxed{(f \circ g)(x) = f(g(x))}$ in detail:



$\boxed{(g \circ f)(x) = g(f(x))}$ in detail:



Example 1: a) Let $f(x) = 2x - 3$ and $g(x) = x^2$

Evaluate $(f \circ g)(3) = f(g(3)) = f(9) = 2 \cdot 9 - 3 = 18 - 3 = 15$

$g(3) = 3^2 = 9$

b) Let $f(x) = 2x - 3$ and $g(x) = x^2$

Give $(f \circ g)(x) = f(g(x)) = f(x^2) = \boxed{2x^2 - 3}$

c) Let $f(x) = 2x - 3$ and $g(x) = x^2$

Evaluate $(g \circ f)(-2) = g(f(-2)) = g(-7) = (-7)^2 = 49$

$f(-2) = 2(-2) - 3 = -4 - 3 = -7$

d) Let $f(x) = 2x-3$ and $g(x) = x^2$

Give $(g \circ f)(x)$ and simplify as much as possible.

$(g \circ f)(x) = g(f(x)) = g(2x-3) = (2x-3)^2 = (2x-3)(2x-3)$

$= 4x^2 - 6x - 6x + 9 = \boxed{4x^2 - 12x + 9}$

Example 2: a) Let $f(x) = \sqrt{x+1}$ and $g(x) = x^2 - 5$

Give $(f \circ g)(x)$ and simplify.

$(f \circ g)(x) = f(g(x)) = f(x^2-5) = \sqrt{x^2-5+1} = \boxed{\sqrt{x^2-4}}$

Same as

$\sqrt{(x+2)(x-2)}$

b) Let $f(x) = \sqrt{x+1}$ and $g(x) = x^2 - 5$

Give $(g \circ f)(x)$ and simplify as much as possible.

$(g \circ f)(x) = g(f(x)) = g(\sqrt{x+1}) = (\sqrt{x+1})^2 - 5 = x+1-5 = \boxed{x-4}$