MAT 012 Lecture Notes, Ch 12.2: Inverse Functions

<u>Inverse Functions</u>:

<u>Note</u>: The little "negative one" does NOT mean "one over". It is just a name for the inverse.

Notation of an inverse function:

For the original function $f(x)$ its inverse function is denoted by [ ]

An inverse function **undoes** the effect of the function:

Example: for $f(x) = x + 7$ is $f^{-1}(x) =$ [ ]

for $f(x) = 2x$ is $f^{-1}(x) =$ [ ]

for $f(x) = \sqrt{x}$ is $f^{-1}(x) =$ [ ]

Finding the inverse of a function algebraically (if we know it has an inverse):

1. Write $f(x)$ as *y*.
2. Swap all *x* and *y*. (This is now a new function, not $f(x)$ any longer)
3. Solve the equation for *y*.
4. Replace *y* with $f^{-1}(x)$ (because you have really discovered the inverse function).

Example: Let $f(x) = 6x + 2$. Determine its inverse function.

Example: Let $f(x) = 3x - 3$. Determine its inverse function.

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An inverse function

- Interchanges the role of x and y.
- Has to be a function. This means that the original function, which “produces” the inverse, needs to be one-to-one (see below).

One-to-one function

A function is one-to-one if no y-value is reused (each x has a different y).

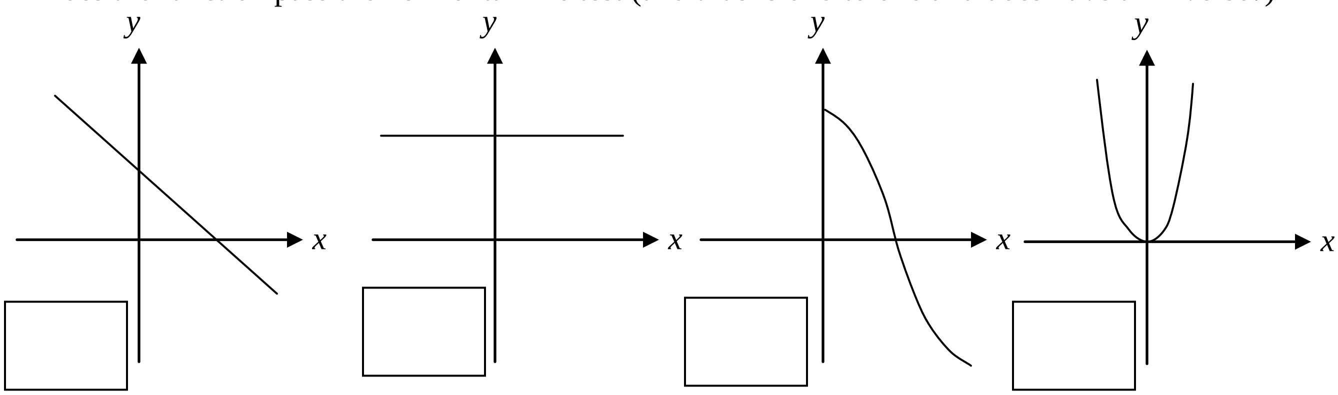
Being one-to-one guarantees that the inverse of $f(x)$ is also a function.

Checking if a function is one-to-one

The original function $f(x)$ has to pass the horizontal line test, where each horizontal line intersects the graph <u>not more</u> than once!

Horizontal Line Test

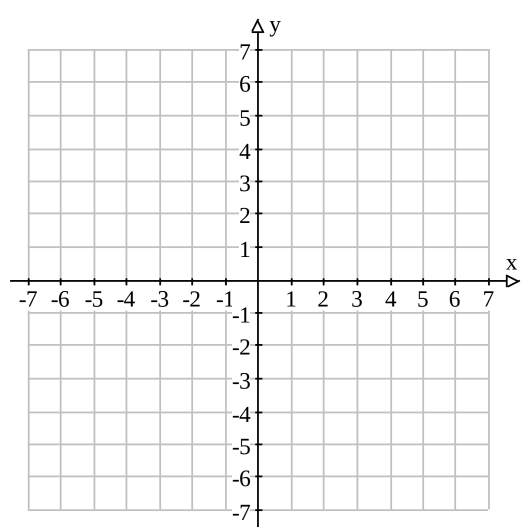
Does the function pass the horizontal line test (and thus is one-to one and does have an inverse?)



Graphical representation of inverse functions:

Example: $f(x) = 3x - 3$

And its inverse $f^{-1}(x) =$



So the inverse function is a reflection of the graph across the line $y = x$ (which is a line at a 45^{O} angle in quadrants I and III and goes through the origin).