MAT 012 **Lecture Notes, Ch 12.2: Inverse Functions**

Inverse Functions:

Note: The little "negative one" does NOT mean "one over". It is just a name for the inverse.

Notation of an inverse function:

For the original function $f(x)$ its inverse function is denoted by $f^{-1}(x)$

An inverse function **undoes** the effect of the function:

Example: for $f(x) = x+7$ is $f^{-1}(x) =$ $x - 7$

for $f(x) = 2x$ is $f^{-1}(x) =$ $\frac{1}{2}x$ or $f^{-1}(x) = \frac{x}{2}$

for $f(x) = \sqrt{x}$ is $f^{-1}(x) =$ x^2

Finding the inverse of a function algebraically (if we know it has an inverse):

1. Write $f(x)$ as y.
2. Swap all x and y. (This is now a new function, not $f(x)$ any longer)
3. Solve the equation for y.
4. Replace y with $f^{-1}(x)$ (because you have really discovered the inverse function).

Example: Let $f(x) = 6x+2$. Determine its inverse function.

$y = 6x+2$ ← original function

$x = 6y+2$ ← inverse function

$-2 \quad -2$

$\frac{x-2}{6} = \frac{6y}{6}$

$y = \frac{x-2}{6}$ $\quad f^{-1}(x) = \frac{x-2}{6}$ alternatively: $f^{-1}(x) = \frac{1}{6}x - \frac{1}{3}$

Example: Let $f(x) = 3x-3$. Determine its inverse function.

$y = 3x - 3$

$x = 3y - 3$ ← inverse

$+3 \quad +3$

$\frac{x+3}{3} = \frac{3y}{3}$

$\frac{x+3}{3} = y$ $\quad f^{-1}(x) = \frac{x+3}{3}$ alternatively: $f^{-1}(x) = \frac{1}{3}x + 1$

An inverse function

- Interchanges the role of x and y.
- Has to be a function. This means that the original function, which "produces" the inverse, needs to be one-to-one (see below).

One-to-one function

A function is one-to-one if no y-value is reused (each x has a different y).

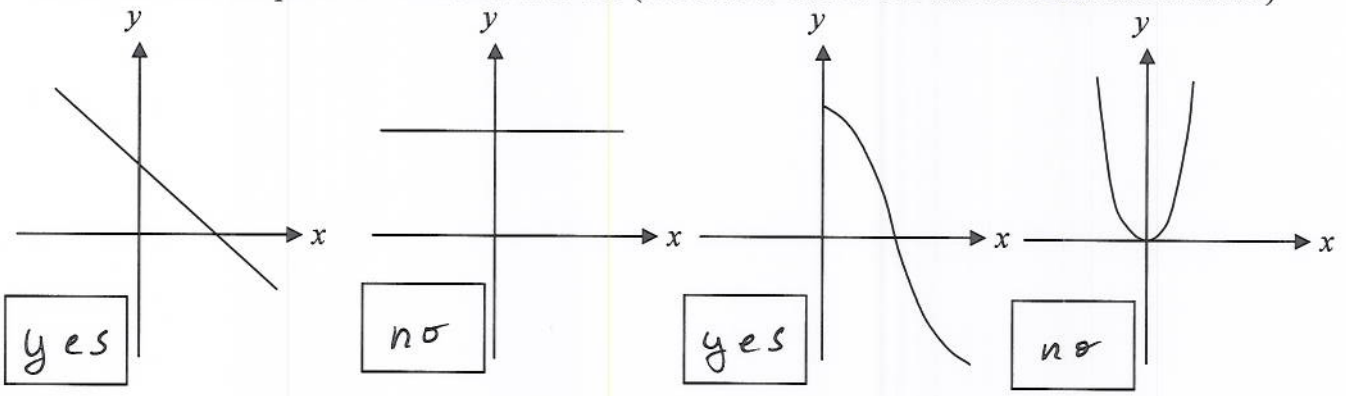
Being one-to-one guarantees that the inverse of $f(x)$ is also a function.

Checking if a function is one-to-one

The original function $f(x)$ has to pass the horizontal line test, where each horizontal line intersects the graph not more than once!

Horizontal Line Test

Does the function pass the horizontal line test (and thus is one-to one and does have an inverse?)



Graphical representation of inverse functions:

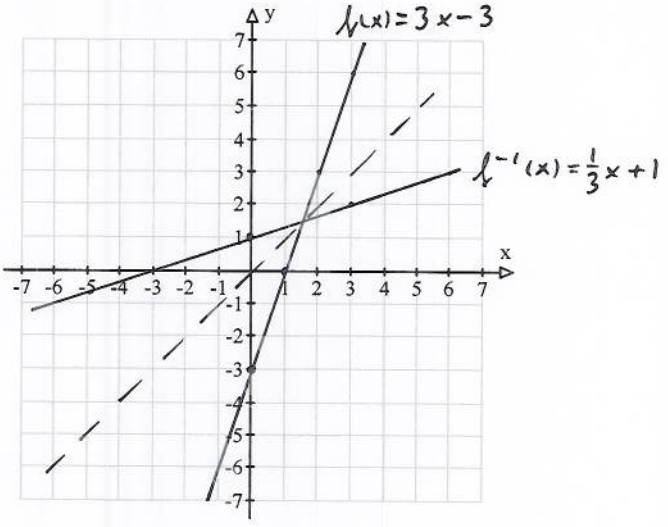
Example: $f(x) = 3x - 3$

And its inverse $f^{-1}(x) = \frac{1}{3}x + 1$ (see last page)

x	$f(x)$
0	−3
1	0
2	3

x	$f^{-1}(x)$
−3	0
0	1
3	2

table of $f(x)$ reversed
$x \to y$; $y \to x$



So the inverse function is a reflection of the graph across the line $y = x$ (which is a line at a 45^{O} angle in quadrants I and III and goes through the origin).