MAT 012 **Lecture Notes: Ch 8.4, Variation**

DIRECT VARIATION:

$\boxed{y = kx}$ where $k > 0$ is a positive constant and $x > 0$

we say: "y varies directly as x"
or "y is (directly) proportional to x"

Note: As one quantity increases the other quantity also increases by the same factor

Procedure:

- Use the given point (the given x value and its respective y value) to find k, i.e. substitute x and y into $y = kx$ and solve for k.
- Use the function to evaluate the missing variable, i.e. substituting k and the last given fact.

Example: The amount of a certain drug (theophylline) given to patients is directly proportional to the person's weight in pounds. This measurement system is used in children as well as adults and 200mg of the drug is given to a boy weighing 80 lbs.

a) Find an equation that models this variation. (*Start with a legend for the variables used.*)

y – variable: Drug dosage, d (in mg)

x – variable: Weight of person (in lbs)

$y = kx$, here $d = kw$ $\quad d = 200$ mg, $\quad w = 80$ lbs

$$\frac{200}{80} = \frac{k \cdot \cancel{80}}{\cancel{80}}$$

$$\frac{200}{80} = k$$

$$2.5 = k \qquad \boxed{d = 2.5w}$$

b) Use your equation from above to determine how much of the drug should be administered to a female patient who weighs 124 lbs.

$w = 124$ lbs, $\quad d = 2.5w$,

so $d = 2.5 \cdot 124$

$d = 310$ mg $\quad$ The drug dosage is 310 mg for this patient.

c) Use your equation from above to determine how much a person weighs who receives 420 mg of the drug.

$d = 420$ mg, $\quad d = 2.5w$,

so $$\frac{420}{2.5} = \frac{\cancel{2.5}w}{\cancel{2.5}}$$

$168 = w$

The weight of this person is 168 lbs.

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INVERSE VARIATION:

$\boxed{y = \frac{k}{x}}$ where $k > 0$ is a positive constant and $x > 0$

we say: "y varies inversely as x"
or "y is inversely proportional to x"

<u>Note</u>: As one quantity increases the other quantity decreases (but NOT at a constant rate).

Procedure:

- Use the given point (the given x value and its respective y value) to find k, i.e. substitute x and y into $y = \frac{k}{x}$ and solve for k.
- Use the function to evaluate the missing variable, i.e. substituting k and the last given fact.

Example: The amount of time, t, it takes a block of ice to melt in water is inversely proportional to the water temperature F in degrees Fahrenheit. In 75^{O} F water, a block of ice of a certain size takes 10 minutes to melt.

a) Find an equation that models this inverse variation.(*Start with a legend for the variables used.*)

y-variable : time, t (in min)
x-variable : temperature, F (in °F)

$y = \frac{k}{x}$, here $t = \frac{k}{F}$ $t = 10$ min, $F = 75°F$

$$10 = \frac{k}{75}$$

$$\frac{10}{1} = \frac{k}{75}$$

$$k = 10 \cdot 75$$

$$k = 750$$

$$\boxed{t = \frac{750}{F}}$$

b) Use your equation from above to determine what temperature the water has when a block of ice of the same size is melting in 12 minutes.

$t = 12$ min

$$t = \frac{750}{F}$$

$$12 = \frac{750}{F}$$

$$\frac{12}{1} = \frac{750}{F}$$

$$\frac{12F}{12} = \frac{750}{12}$$

$$F = \frac{750}{12}$$

$$F = 62.5\ °F$$

The temperature is 62.5°F.

COMBINED variation:

In combined variation, only one constant k is necessary.

- Variation can include **squares**
  Example: Let y be directly proportional to the square of z. Find the equation for the variation. $y = kz^2$
- **Joint** variation (there is more than one x-value)
  Example: y is a joint direct variation of t and r. Find the equation for the variation.
  $$y = ktr$$
- Combinations made of direct as well as inverse variation are also possible.
  Example: y varies directly as g and inversely as the square of u. Find the equation for the variation.
  $$y = k\frac{g}{u^2} \quad \text{same as} \quad y = \frac{kg}{u^2}$$

Example: When a wind blows perpendicularly against a flat surface (sail), its force F is jointly proportional to the surface area A and the speed of the wind S. A sail whose surface area is 12 square feet experiences a 20 pound force when the wind speed is 10 mph.

a) Find an equation that models this variation. (*Start with a legend for the variables used.*)

y-variable : force, F (in lbs)
x-variables : surface area, A (in ft^2)
windspeed, S (in mph)

$F = kAS \quad A = 12\ ft^2, \ S = 10\ mph, \ F = 20\ lbs$

$20 = k \cdot 12 \cdot 10$

$\frac{20}{120} = \frac{120k}{120}$

$\frac{1}{6} = k \qquad F = \frac{1}{6}AS$

b) Use the equation to find the force on an 40-square-foot sail if the wind speed is 12 mph.

$A = 40\ ft^2, \ S = 12\ mph, \ F = \frac{1}{6}AS$

$F = \frac{1}{6} \cdot 40 \cdot 12 = 80\ lbs$

The force is 80 lbs.

c) Use the equation to find the wind speed when an 18 square-foot sail creates a force of 66 lbs.

$A = 18\ ft^2, \ F = 66\ lbs \qquad F = \frac{1}{6}AS$

$66 = \frac{1}{6} \cdot 18 \cdot S$

$\frac{66}{3} = \frac{3S}{3}$

$22 = S$

The windspeed is 22 mph.

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Extra Practice

1) An alligator's tail length, T, varies directly as its body length, B. An alligator with a body length of 4 ft has a tail length of 3.6 ft.
   a) Find an equation that models this variation. (*Start with a legend for the variables used.*)

   y-variable: Alligator's tail length, T (in ft)
   x-variable: Alligator's body length, B (in ft)
   $y = kx$, here $T = kB$

   $$\frac{3.6}{4} = \frac{k \cdot 4}{4}$$
   $0.9 = k$ $\quad T = 0.9B$

   b) What is the tail length of an alligator whose body length is 6 ft?

   $T = 0.9 \cdot 6$
   $T = 5.4$ ft The tail length is 5.4 ft

2) A mammal's average life span, L, in years varies inversely as its average heart rate, R, in beats per minute. Cats have an average life span of 15 years and an average heart rate of 126 beats per minute.
   a) Find an equation that models this variation. (*Start with a legend for the variables used.*)

   y-variable: Mammal's average life span, L (in years)
   x-variable: Mammal's average heart rate, R (in bpm)
   $y = \frac{k}{x}$, here $L = \frac{k}{R}$

   $$15 = \frac{k}{126}$$
   $15 \cdot 126 = k$ $\quad 1890 = k$ $\quad L = \frac{1890}{R}$

   b) Mice have an average life span of 3 years. Determine their average heart rate.

   $L = 3$ years $\quad 3 = \frac{1890}{R}$

   $\frac{3}{1} = \frac{1890}{R}$ $\quad \frac{3R}{3} = \frac{1890}{3}$ $\quad R = 630$ bpm

   The average heart rate of a mouse is 630 bpm.

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ANSWERS to Extra Practice

1) a) $T = 0.9 \cdot B$ (B: Alligator body length in ft; T: Alligator tail length in ft)
   b) The tail length is 5.4 ft.

2) a) $L = \frac{1890}{R}$ (R: Mammals heart rate in beats per minute; L: Mammals life span in years)
   b) Average heart rate of mice: 630 beats per minute.