SOLUTIONS

MAT 012 **PRACTICE TEST 3**

Prof. Clayton Will not be collected, but will help you prepare for the test.

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For the "roots" and "radicals", we are assuming that all variables/variable expressions stand for positive values unless the instructions specifically state that the variables/variable expressions can assume ANY real value.

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1. Find the two solutions of the equation $|2x-5|-3=10$

$$+3 \quad +3$$

$$|2x-5| = 13$$

$$2x-5=-13 \quad \text{or} \quad 2x-5=13$$

$$+5 \quad +5 \qquad\qquad +5 \quad +5$$

$$\frac{2x}{2} = \frac{-8}{2} \qquad\qquad \frac{2x}{2} = \frac{18}{2}$$

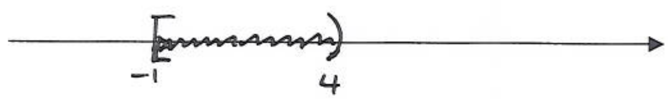
$$x=-4 \quad \cup \quad x=9$$

2. Solve, then **graph** the solutions **on the number line**, also give the answers in interval notation. $1 \le 3x+4 < 16$

$$-4 \quad -4 \quad -4$$

$$\frac{-3}{3} \le \frac{3x}{3} < \frac{12}{3}$$

$$-1 \le x < 4$$



In **interval notation**:

$[-1, 4)$

3. Give in interval notation:

$(-\infty, -4] \cup (3, \infty)$

4. Solve, then graph the solutions on a number line and state them in interval notation:

$$|4x-9|+2>13$$

$$-2 \quad -2$$

$$|4x-9| > 11$$

$$4x-9<-11 \quad \text{or} \quad 4x-9>11$$

$$+9 \quad +9 \qquad\qquad +9 \quad +9$$

$$\frac{4x}{4} < \frac{-2}{4} \qquad\qquad \frac{4x}{4} > \frac{20}{4}$$

$$x < -\tfrac{1}{2} \quad \cup \quad x > 5$$

Interval notation:

$(-\infty, -\tfrac{1}{2}) \cup (5, \infty)$

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5. Solve, then graph the solutions on a number line and state them in interval notation:

$$\frac{-4|2x-7|}{-4} \geq \frac{-12}{-4}$$

$$|2x-7| \leq 3$$

$$-3 \leq 2x-7 \leq 3$$
$$+7 \qquad +7 \qquad +7$$

$$\frac{4}{2} \leq \frac{2x}{2} \leq \frac{10}{2}$$

$$2 \leq x \leq 5$$

Interval notation: $[2, 5]$

6. Solve $|3x-2| < -7$

positive < negative

always FALSE

no solutions

4. Solve $|6x-8| > -10$

positive > negative

always TRUE

All real numbers, $\mathbb{R}$

Same as $(-\infty, \infty)$

5. Give the solutions to $x^2 = 25$

$x = -5, \ x = 5$

6. Simplify: $\sqrt{49x^6} = 7x^3$

7. Simplify: $\sqrt[20]{a^{15}b^5} = a^{\frac{15}{20}} b^{\frac{5}{20}} = a^{\frac{3}{4}} b^{\frac{1}{4}} = \sqrt[4]{a^3 b}$

8. Simplify: $\sqrt[3]{-64x^3} = -4x$

9. Evaluate $\left(\frac{1}{81}\right)^{-\frac{1}{4}} = \left(\frac{81}{1}\right)^{\frac{1}{4}} = \frac{\sqrt[4]{81}}{\sqrt[4]{1}} = \frac{\sqrt[4]{81}}{1} = \sqrt[4]{81} = 3$

10. Convert to an equivalent expression with no negative exponents. Then write in radical notation: $a^{\frac{4}{3}} b^{-\frac{1}{2}} = \frac{a^{\frac{4}{3}}}{b^{\frac{1}{2}}} = \underbrace{\frac{\sqrt[3]{a^4}}{\sqrt{b}}}_{\text{o.k.}} = \frac{\sqrt[3]{a^3 a}}{\sqrt{b}} = \underbrace{\frac{a\sqrt[3]{a}}{\sqrt{b}}}_{\text{simplified version}}$

11. Convert to an equivalent expression with no negative exponents. Simplify and write in radical notation: $\frac{a^{\frac{2}{5}} c^{-\frac{1}{3}} b^{\frac{3}{4}}}{a^{-\frac{1}{5}}} = \frac{a^{\frac{2}{5}} a^{\frac{1}{5}} b^{\frac{3}{4}}}{c^{\frac{1}{3}}} = \frac{a^{\frac{3}{5}} b^{\frac{3}{4}}}{c^{\frac{1}{3}}} = \frac{\sqrt[5]{a^3}\ \sqrt[4]{b^3}}{\sqrt[3]{c}}$

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12. Combine, simplify and give the answer in radical notation: $\sqrt[4]{x}\cdot\sqrt[3]{x^2} =$

$$\sqrt[4]{x}\cdot\sqrt[3]{x^2} = x^{\frac{1}{4}}\,x^{\frac{2}{3}} = x^{\frac{1}{4}+\frac{2}{3}} = x^{\frac{1\cdot3}{4\cdot3}+\frac{2\cdot4}{3\cdot4}} = x^{\frac{3}{12}+\frac{8}{12}} = x^{\frac{11}{12}} = \sqrt[12]{x^{11}}$$

13. Write in fractional exponent notation: $\sqrt[5]{(4x)^3} =$ $(4x)^{\frac{3}{5}}$

14. Simplify the numerator. Then write with no negative powers and convert to radical notation:

$$\frac{125^{\frac{1}{3}}}{x^{-\frac{1}{2}}} = \frac{\sqrt[3]{125}}{x^{-\frac{1}{2}}} = \frac{5}{x^{-\frac{1}{2}}} = 5x^{\frac{1}{2}} = 5\sqrt{x}$$

15. Simplify: $\sqrt[4]{\sqrt{2x}} = \sqrt[8]{2x}$ Alternatively: $\sqrt[4]{\sqrt{2x}} = \left((2x)^{\frac{1}{2}}\right)^{\frac{1}{4}} = (2x)^{\frac{1}{2}\cdot\frac{1}{4}} = (2x)^{\frac{1}{8}} = \sqrt[8]{2x}$

16. Simplify: $\sqrt[5]{\sqrt[3]{x}} = \sqrt[15]{x}$ Alternatively: $\sqrt[5]{\sqrt[3]{x}} = \left(x^{\frac{1}{3}}\right)^{\frac{1}{5}} = x^{\frac{1}{3}\cdot\frac{1}{5}} = x^{\frac{1}{15}} = \sqrt[15]{x}$

17. Simplify: $\sqrt{720} = \sqrt{144\cdot5} = 12\sqrt{5}$

18. Simplify: $\sqrt[6]{4096\,x^{18}\,y^{12}} = 4x^3y^2$

19. Simplify: $\sqrt{810\,a^{11}\,b^6} = \sqrt{81\cdot10\,a^{10}a\,b^6} = 9a^5b^3\sqrt{10a}$

20. Simplify $\sqrt[4]{81x^9y^{15}} = \sqrt[4]{81x^8x\,y^{12}y^3} = 3x^2y^3\sqrt[4]{xy^3}$

21. Combine and simplify as much as possible: $\sqrt{3x^5}\cdot\sqrt{6x^4} = \sqrt{18x^9} = \sqrt{9\cdot2\,x^8x} = 3x^4\sqrt{2x}$

22. Combine, then simplify as much as possible:

$$\sqrt[3]{-4x^5y^7}\cdot\sqrt[3]{54x^5y} = \sqrt[3]{-216x^{10}y^8} = \sqrt[3]{-216\,x^9x\,y^6y^2} = -6x^3y^2\sqrt[3]{xy^2}$$

23. Combine, then simplify as much as possible:

$$\sqrt{\frac{5a^2}{4b}}\cdot\sqrt{\frac{15b^4}{2a^2}} = \sqrt{\frac{75\cancel{a^2}b^4}{8\cancel{a^2}b}} = \sqrt{\frac{75b^3}{8}} = \sqrt{\frac{25\cdot3\,b^2b}{4\cdot2}} = \frac{5b}{2}\sqrt{\frac{3b}{2}}$$

24. Simplify:

$$\sqrt[5]{\frac{32\,y^{10}x^{-10}}{81\,z^{-3}}}\cdot\sqrt[5]{\frac{3^{-1}\,x^{-10}}{y^{-5}\,z^{3}}} = \sqrt[5]{\frac{32\,y^{10}\,z^{3}}{81\,x^{10}}}\cdot\sqrt[5]{\frac{y^{5}}{3\,x^{10}z^{3}}} = \sqrt[5]{\frac{32\,y^{15}\,\cancel{z^{3}}}{243\,x^{20}\,\cancel{z^{3}}}} = \frac{2y^{3}}{3x^{4}}$$

25. Simplify $3\sqrt{8}+\sqrt{32}-4\sqrt{50}$

$$= 3\sqrt{4\cdot 2} + \sqrt{16\cdot 2} - 4\sqrt{25\cdot 2}$$

$$= 3\cdot 2\sqrt{2} + 4\sqrt{2} - 4\cdot 5\sqrt{2}$$

$$= 6\sqrt{2} + 4\sqrt{2} - 20\sqrt{2}$$

$$= -10\sqrt{2}$$

26. Simplify $2\sqrt{180}+\sqrt{12}-4\sqrt{48}-\sqrt{45}$

$$= 2\sqrt{36\cdot 5} + \sqrt{4\cdot 3} - 4\sqrt{16\cdot 3} - \sqrt{9\cdot 5}$$

$$= 2\cdot 6\sqrt{5} + 2\sqrt{3} - 4\cdot 4\sqrt{3} - 3\sqrt{5}$$

$$= 12\sqrt{5} + 2\sqrt{3} - 16\sqrt{3} - 3\sqrt{5}$$

$$= 9\sqrt{5} - 14\sqrt{3}$$

27. Find the distance between $(\overset{x_1}{-4}, \overset{y_1}{10})$ and $(\overset{x_2}{2}, \overset{y_2}{6})$.

Give the answer exact ***and then*** *a decimal approximation (rounded to one decimal place.)*

$$d = \sqrt{(y_2-y_1)^2 + (x_2-x_1)^2}$$

$$= \sqrt{(6-10)^2 + (2-(-4))^2}$$

$$= \sqrt{(-4)^2 + (2+4)^2}$$

$$= \sqrt{(-4)^2 + 6^2}$$

$$= \sqrt{16+36}$$

$= \sqrt{52}$ ← exact: $\underbrace{\sqrt{52}}_{\text{o.k.}}$ same as $\sqrt{4\cdot 13} = 2\sqrt{13}$

≈ 7.2 ← decimal approximation

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28. Algebraically, find the midpoint of the line segment from $(-9,\ -7)$ to $(1,\ 3)$ (*Give your answer in ordered pair notation.*)

$x_1 \quad y_1 \qquad x_2 \quad y_2$

$$x_M = \frac{x_2 + x_1}{2} \qquad y_M = \frac{y_2 + y_1}{2}$$

$$x_M = \frac{1+(-9)}{2} = \frac{1-9}{2} = \frac{-8}{2} = -4$$

Midpoint: $(-4, -2)$

$$y_M = \frac{3+(-7)}{2} = \frac{3-7}{2} = \frac{-4}{2} = -2$$

29. Multiply and simplify: $5\sqrt{6}\left(3\sqrt{2}-\sqrt{5}\right)$

$$5\sqrt{6}\left(3\sqrt{2}-\sqrt{5}\right) = 15\sqrt{12} - 5\sqrt{30}$$

30. Multiply and simplify: $\left(5\sqrt{3}-3\sqrt{2}\right)\left(4\sqrt{3}-2\sqrt{7}\right) = 20\cdot 3 - 10\sqrt{21} - 12\sqrt{6} + 6\sqrt{14}$

$$= 60 - 10\sqrt{21} - 12\sqrt{6} + 6\sqrt{14}$$

same as $2\left(30 - 5\sqrt{21} - 6\sqrt{6} + 3\sqrt{14}\right)$

For these last problems, each variable can assume **ANY real value**. (Put absolute value where necessary, but ONLY where necessary.) If the expression is not defined in the real numbers, say so. If the expression cannot be simplified, say so.

31. a) Evaluate $\sqrt[3]{-216} = -6$

b) Simplify $\sqrt{x^{18}} = |x^9|$

c) Simplify $\sqrt[3]{64x^3} = 4x$

d) Simplify $\sqrt{\dfrac{x^2}{y^6}} = \left|\dfrac{x}{y^3}\right|$

e) Simplify $\sqrt[4]{625x^{12}} = |5x^3|$

f) Simplify $-\sqrt{36x^{26}} = -|6x^{13}|$

g) Evaluate $\sqrt[6]{-64}$ = undefined

h) Simplify $\sqrt[7]{x^{35}y^{14}} = x^5y^2$

i) Simplify $-\sqrt[3]{-343x^{15}} =$

$-(-7x^5) = 7x^5$

j) Simplify $\sqrt{x^2+25}$ = Cannot be simplified

k) Simplify $\sqrt{x^2-6x+9} = \sqrt{(x-3)(x-3)}$

$$= \sqrt{(x-3)^2} = |x-3|$$

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32. Simplify: $LCD = x^3$

$$\frac{\frac{4}{x}-\frac{7}{x^2}}{\frac{3}{x^3}-\frac{5}{x}} = \frac{\frac{4}{x}\cdot\frac{x^3}{1}-\frac{7}{x^2}\cdot\frac{x^3}{1}}{\frac{3}{x^3}\cdot\frac{x^3}{1}-\frac{5}{x}\cdot\frac{x^3}{1}} = \frac{\frac{4}{\cancel{x}}\cdot\frac{\cancel{x^3}^{2}}{1}-\frac{7}{\cancel{x^2}}\cdot\frac{\cancel{x^3}}{1}}{\frac{3}{\cancel{x^3}}\cdot\frac{\cancel{x^3}}{1}-\frac{5}{\cancel{x}}\cdot\frac{\cancel{x^3}^{2}}{1}} = \frac{4x^2-7x}{3-5x^2}$$

same as $\frac{x(4x-7)}{3-5x^2}$

33. Simplify (After the initial steps, don't forget to factor and simplify again): $LCD = a^2b^2$

$$\frac{\frac{1}{b^2}-\frac{1}{a^2}}{\frac{1}{b}+\frac{1}{a}} = \frac{\frac{1}{\cancel{b^2}}\cdot\frac{a^2\cancel{b^2}}{1}-\frac{1}{\cancel{a^2}}\cdot\frac{\cancel{a^2}b^2}{1}}{\frac{1}{\cancel{b}}\cdot\frac{a^2\cancel{b^2}}{1}+\frac{1}{\cancel{a}}\cdot\frac{\cancel{a^2}b^2}{1}} = \frac{a^2-b^2}{a^2b+ab^2} = \frac{\cancel{(a+b)}(a-b)}{ab\cancel{(a+b)}}$$

$$= \frac{a-b}{ab}$$

same as $\frac{\cancel{a}^{1}}{\cancel{a}b}-\frac{\cancel{b}^{1}}{a\cancel{b}} = \frac{1}{b}-\frac{1}{a}$