SOLUTIONS

MAT 012 **PRACTICE TEST 4**

Prof. Clayton Will not be collected, but will help you prepare for the test.

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Use complex numbers as appropriate. (All expressions are defined.)

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1. Simplify $\sqrt{-144} = 12i$

2. Add and simplify: $(12+5i)+(3-i) = 12+5i+3-i$
$= 15+4i$

3. Subtract and simplify: $(7-2i)-(10-9i) = 7-2i-10+9i$
$= -3+7i$

4. Multiply and simplify: $(3-4i)(2+3i) = 6+9i-8i-12i^2$
$= 6+i-12(-1)$
$= 6+i+12$
$= 18+i$

5. Simplify: Give the answer in the form $a + bi$.

$$\frac{(2+7i)}{6i} \cdot \frac{i}{i} = \frac{2i+7i^2}{6i^2} = \frac{2i-7}{-6} = \frac{7}{6} - \frac{2}{6}i = \frac{7}{6} - \frac{1}{3}i$$

6. Simplify: Give the answer in the form $a + bi$.

$$\frac{(3-4i)}{(2+5i)} \cdot \frac{(2-5i)}{(2-5i)} = \frac{6-15i-8i+20i^2}{4-10i+10i-25i^2} = \frac{6-23i-20}{4+25} = \frac{-14-23i}{29}$$

$$= -\frac{14}{29} - \frac{23}{29}i$$

7. Rationalize the denominator $\dfrac{9\sqrt{7}}{2\sqrt{6}}$ (*Make sure to give the answer in lowest terms.*)

$$\frac{9\sqrt{7}}{2\sqrt{6}} = \frac{9\sqrt{7}}{2\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{\overset{3}{\cancel{9}}\sqrt{42}}{2\cdot \underset{2}{\cancel{6}}} = \frac{3\sqrt{42}}{4}$$

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8. Rationalize the denominator

$$\frac{(\sqrt{3}-\sqrt{5})}{(\sqrt{5}-\sqrt{2})}\cdot\frac{(\sqrt{5}+\sqrt{2})}{(\sqrt{5}+\sqrt{2})}=\frac{\sqrt{15}+\sqrt{6}-5-\sqrt{10}}{5+\sqrt{10}-\sqrt{10}-2}=\frac{\sqrt{15}+\sqrt{6}-5-\sqrt{10}}{5-2}$$

$$=\frac{\sqrt{15}+\sqrt{6}-5-\sqrt{10}}{3}$$

9. Rationalize the denominator

$$\frac{(4\sqrt{6}-2\sqrt{3})}{(2\sqrt{5}+3\sqrt{2})}\cdot\frac{(2\sqrt{5}-3\sqrt{2})}{(2\sqrt{5}-3\sqrt{2})}=\frac{8\sqrt{30}-12\sqrt{12}-4\sqrt{15}+6\sqrt{6}}{4\cdot5-6\sqrt{10}-6\sqrt{10}-9\cdot2}$$

$$=\frac{8\sqrt{30}-12\sqrt{12}-4\sqrt{15}+6\sqrt{6}}{20-18}=\frac{8\sqrt{30}-12\sqrt{4\cdot3}-4\sqrt{15}+6\sqrt{6}}{2}$$

$$=\frac{\cancel{2}^{1}(4\sqrt{30}-6\cdot2\sqrt{3}-2\sqrt{15}+3\sqrt{6})}{\cancel{2}_{1}}=4\sqrt{30}-12\sqrt{3}-2\sqrt{15}+3\sqrt{6}$$

10. Use the **quadratic formula** to solve: $3x^2=7x-1$

(*Give an* ***exact*** *answer, not a decimal approximation.*)

$$3x^2=7x-1$$
$$-7x+1 \quad -7x+1$$
$$3x^2-7x+1=0$$

$a=3$

$b=-7$

$c=1$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{7\pm\sqrt{(-7)^2-4\cdot3\cdot1}}{6}$$

$$=\frac{7\pm\sqrt{49-12}}{6}=\frac{7\pm\sqrt{37}}{6}$$

11. Use the **quadratic formula** to solve: $5x^2-2=4x$

(*Give an approximate answer rounded to two decimal places*)

$$5x^2-2=4x$$
$$-4x \quad -4x$$
$$5x^2-4x-2=0$$

$a=5$

$b=-4$

$c=-2$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{4\pm\sqrt{(-4)^2-4\cdot5\cdot(-2)}}{10}=\frac{4\pm\sqrt{16+40}}{10}$$

$$=\frac{4\pm\sqrt{56}}{10}$$

$$x_1=\frac{4+\sqrt{56}}{10}\approx1.15,\quad x_2=\frac{4-\sqrt{56}}{10}\approx-0.35$$

(same as $\frac{2\pm\sqrt{14}}{5}$ but not needed because of decimal asked for)

12. a) Find the domain for the function $f(x) = \sqrt{2x+6}$

$$2x + 6 \geq 0$$
$$-6 \quad -6$$
$$\frac{2x}{2} \geq \frac{-6}{2}$$
$$x \geq -3$$

b) Solve $1 + \sqrt{2x+6} = x$

$$-1 \qquad -1$$
$$\sqrt{2x+6} = x-1$$
$$\left(\sqrt{2x+6}\right)^2 = (x-1)^2$$
$$2x+6 = (x-1)(x-1)$$
$$2x+6 = x^2 - x - x + 1$$
$$2x+6 = x^2 - 2x + 1$$
$$-2x - 6 \qquad -2x - 6$$
$$0 = x^2 - 4x - 5$$
$$0 = (x-5)(x+1)$$
$$\boxed{x = 5} \text{ or } \cancel{x = -1}$$

c) Show the "checks" to identify extraneous solutions.

check $x = -1$: in $\sqrt{2x+6} = x - 1$

$$\sqrt{-2+6} \stackrel{?}{=} -1-1$$
$$\sqrt{4} \stackrel{?}{=} -2$$
$$2 \neq -2$$

check $x = 5$: in $\sqrt{2x+6} = x - 1$

$$\sqrt{10+6} \stackrel{?}{=} 5-1$$
$$\sqrt{16} \stackrel{?}{=} 4$$
$$4 = 4 \checkmark$$

d) List the solution(s).

$x = 5$

13. Let $\boxed{f(x) = -2x^2 - 12x - 10}$ *(If you run out of space on any part, use notebook paper)*

a) Calculate the vertex (Give the **point**):

$h = \frac{-b}{2a} = \frac{12}{-4} = -3$

$k = f(-3) = -2(-3)^2 - 12(-3) - 10 = -2 \cdot 9 + 36 - 10 = -18 + 36 - 10 = 8$

vertex $(-3, 8)$

b) Give the **equation** of the axis of symmetry: $x = -3$

c) Give the **vertex-form** of the function:

$f(x) = -2(x - (-3))^2 + 8 = -2(x+3)^2 + 8$

d) Give the y-intercept (give the **point**): $(0, -10)$

e) Calculate the x-intercepts (Give the **points**)

$-2x^2 - 12x - 10 = 0$ divide by -2

$x^2 + 6x + 5 = 0$

$(x+5)(x+1) = 0$

$x = -5 \quad x = -1$

points: $(-5, 0)$; $(-1, 0)$

f) Graph as accurately as possible:

by symmetry: $(-6, -10)$

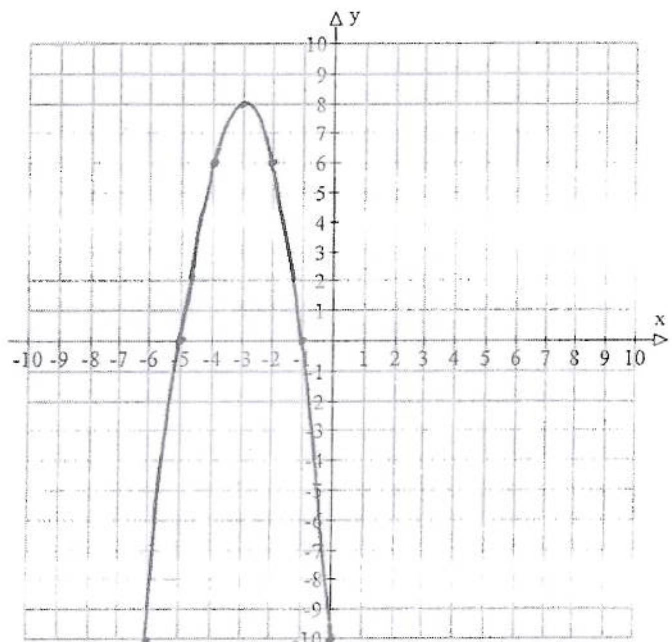
$f(-2) = -2 \cdot (-2)^2 - 12 \cdot (-2) - 10$

$= -2 \cdot 4 + 24 - 10$

$= -8 + 24 - 10 = 6$

point: $(-2, 6)$

by symmetry: $(-4, 6)$



g) This parabola opens downward. This vertex is a maximum.

To obtain this function, x^2 is stretched by the "stretch-factor" 2.

The parabola is shifted 3 units to the left (horizontal shift).

and 8 units up (vertical shift).

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14. Let $\boxed{f(x)=\frac{1}{2}x^2-x-\frac{3}{2}}$ $a=\frac{1}{2}$, $b=-1$, $c=-\frac{3}{2}$

a) Calculate the vertex (Give the **point**): *Show work. Do NOT just find on your calculator.*

$h=\frac{-b}{2a}=\frac{1}{2\cdot\frac{1}{2}}=\frac{1}{1}=1$

$k=f(1)=\frac{1}{2}\cdot 1^2-1-\frac{3}{2}=\frac{1}{2}-1-\frac{3}{2}=-2$ vertex: $(1,-2)$

b) equation of the axis of symmetry: $x=1$

c) vertex-form: $f(x)=a(x-h)^2+k$

$f(x)=\frac{1}{2}(x-1)^2-3$

d) y-intercept (Give the point):

$f(0)=-\frac{3}{2}$ $(0,-\frac{3}{2})$

e) x-intercepts (Give the points if they exist):

$0=\frac{1}{2}x^2-x-\frac{3}{2}$ $a=\frac{1}{2}$, $b=-1$, $c=-\frac{3}{2}$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{1\pm\sqrt{(-1)^2-4\cdot\frac{1}{2}\cdot(-\frac{3}{2})}}{2\cdot\frac{1}{2}}=\frac{1\pm\sqrt{1+3}}{1}=1\pm\sqrt{4}=1\pm 2$

$x_1=1+2=3$ $x_2=1-2=-1$

$(3,0)$ $(-1,0)$

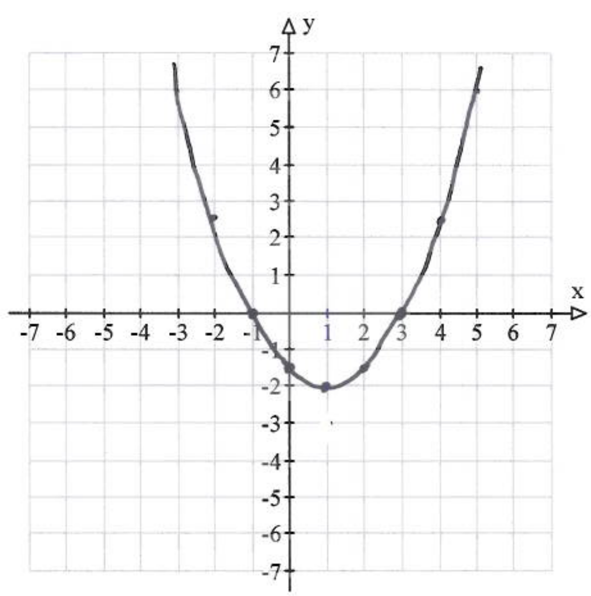
f) Graph as accurately as possible:

Use at least 5 key-points.
(Showing algebraic work on this part is optional.)

by symmetry $(2,-\frac{3}{2})$

$f(4)=\frac{1}{2}\cdot 16-4-\frac{3}{2}=8-4-\frac{3}{2}=2\frac{1}{2}$

by symmetry $(-2,2\frac{1}{2})$



g) This parabola opens <u>upward</u>. This vertex is a <u>minimum</u>.

To obtain this function, x^2 is <u>squeezed (widened)</u> by the "stretch-factor" <u>$\frac{1}{2}$</u>.

The parabola is shifted <u>1</u> units to the <u>right</u> (horizontal shift).

and <u>2</u> units <u>down</u> (vertical shift)

15. $\boxed{f(x)=0.6(x-7)^2+2}$

a) This parabola opens upward. This vertex is a minimum

To obtain this function, x^2 is Squeezed/widened by the "stretch-factor" 0.6.

The parabola is shifted 7 units to the right (horizontal shift).

and 2 units up (vertical shift).

The vertex of the parabola is the point (7, 2).

16. $\boxed{f(x)=3(x+2)^2-14}$

a) This parabola opens upward. This vertex is a minimum

To obtain this function, x^2 is Stretched/narrowed by the "stretch-factor" 3.

The parabola is shifted 2 units to the left (horizontal shift).

and 14 units down (vertical shift).

The vertex of the parabola is the point (-2, -14).

b) Convert the given vertex form to the standard form $f(x)=ax^2+bx+c$.

$$
\begin{aligned}
f(x) &= 3(x+2)^2-14 \\
&= 3[(x+2)(x+2)]-14 \\
&= 3(x^2+2x+2x+4)-14 \\
&= 3(x^2+4x+4)-14 \\
&= 3x^2+12x+12-14 \\
&= 3x^2+12x-2
\end{aligned}
$$

17. Let $f(x)=x^2-5$ *and* $g(x)=4x-2$

(a) Find $(f\circ g)(x)$ and simplify.

$$
\begin{aligned}
(f\circ g)(x) &= f(g(x)) \\
&= (4x-2)^2-5 \\
&= (4x-2)(4x-2)-5 \\
&= 16x^2-8x-8x+4-5 \\
&= 16x^2-16x-1
\end{aligned}
$$

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Let $f(x)=x^2-5$ *and* $g(x)=4x-2$.

(b) Find $(g\circ f)(x)$ and simplify.

$$(g\circ f)(x) = g(f(x)) = 4(x^2-5)-2 = 4x^2-20-2$$
$$= 4x^2-22$$

18. Let $f(x)=2x-7$ and $g(x)=\sqrt{3-x}$ ← -1 $g(-1)=\sqrt{3+1}=\sqrt{4}=2$

a) Give $(f\circ g)(-1)$ and simplify.

$$(f\circ g)(-1) = f(g(-1)) = f(2) = 2\cdot 2-7 = 4-7$$
$$= -3$$

Let $f(x)=2x-7$ and $g(x)=\sqrt{3-x}$

b) Give $(g\circ f)(x)$ and simplify.

$$(g\circ f)(x) = g(f(x)) = \sqrt{3-(2x-7)} = \sqrt{3-2x+7} = \sqrt{10-2x}$$

19. Find the **inverse function** of $f(x)=2x-12$. Use appropriate notation to state the inverse function.

$y = 2x-12$ ← original

$x = 2y-12$ ← inverse

$\underline{+12 \qquad +12}$

$\frac{x+12}{2} = \frac{2y}{2}$

$y = \frac{x+12}{2}$ same as $y = \frac{1}{2}x+6$

$f^{-1}(x) = \frac{x+12}{2}$ same as $f^{-1}(x) = \frac{1}{2}x+6$